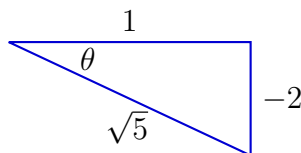


MATH 125 EXAM #4 KEY (FALL 2015)

1 Find $\theta \in [0, \pi]$ such that $\theta = \cos^{-1}[\cos(-\frac{5\pi}{3})]$, or equivalently $\cos \theta = \cos(-\frac{5\pi}{3})$. The only answer is $\theta = \frac{\pi}{3}$.

Next, let $\theta = \tan^{-1}(-2)$, so $\theta \in (-\frac{\pi}{2}, \frac{\pi}{2})$ is such that $\tan \theta = -2$. Thus we have a triangle in Quadrant IV:



From this we can see that $\csc[\tan^{-1}(-2)] = \csc \theta = -\frac{\sqrt{5}}{2}$.

2 The inverse function is

$$f^{-1}(x) = \tan^{-1}\left(\frac{x+3}{2}\right).$$

Range of f : $(-\infty, \infty)$. Domain of f^{-1} : $(-\infty, \infty)$. Range of f^{-1} : $(-\frac{\pi}{2}, \frac{\pi}{2})$.

3a From $\cos^2 \theta = \frac{3}{4}$ we get $\cos \theta = \pm \frac{\sqrt{3}}{2}$. Solutions: $\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$.

3b From $\cot \theta = -\frac{1}{\sqrt{3}}$ we get $\theta = \frac{2\pi}{3}, \frac{5\pi}{3}$.

3c Either $\cot \theta = -1$ or $\csc \theta = \frac{1}{2}$. The latter implies that $\sin \theta = 2$, which has no solution. Thus the only solutions are: $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$.

3d We have

$$\tan \theta = 2 \sin \theta \Rightarrow \frac{\sin \theta}{\cos \theta} - 2 \sin \theta = 0 \Rightarrow (\sin \theta)(\sec \theta - 2) = 0.$$

Solutions: $\theta = 0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}$.

4a We have

$$\tan^3 x + \tan x = (\tan x)(\tan^2 x + 1) = (\tan x)(\sec^2 x) = \sec^2 x \tan x.$$

4b We have

$$\begin{aligned} 1 - \frac{\sin^2 u}{1 + \cos u} &= \frac{(1 + \cos u) - \sin^2 u}{1 + \cos u} = \frac{(1 + \cos u) - (1 - \cos^2 u)}{1 + \cos u} \\ &= \frac{\cos^2 u + \cos u}{1 + \cos u} = \frac{(\cos u)(\cos u + 1)}{1 + \cos u} = \cos u. \end{aligned}$$

5 With the identity $\sin(u + v) = \sin u \cos v + \cos u \sin v$,

$$\sin \frac{\pi}{18} \cos \frac{5\pi}{18} + \cos \frac{\pi}{18} \sin \frac{5\pi}{18} = \sin \left(\frac{\pi}{18} + \frac{5\pi}{18} \right) = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}.$$

6 With a half-angle identity,

$$\sin 195^\circ = -\sqrt{\frac{1 - \cos 390^\circ}{2}} = -\sqrt{\frac{1 - \cos 30^\circ}{2}} = -\sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}} = -\frac{\sqrt{2 - \sqrt{3}}}{2}.$$

7 Angles are 90° , $\sin^{-1}(1/3) \approx 19.47^\circ$, and $90^\circ - \sin^{-1}(1/3) \approx 70.53^\circ$.

8 Let h be the height of the monument. Then

$$\tan 35.1^\circ = \frac{x}{789} \Rightarrow x = 789 \tan 35.1^\circ \approx 554.5 \text{ ft}$$

9a By the Law of Sines:

$$\sin A = \frac{a \sin C}{c} = \frac{56.2 \sin(46^\circ 32')}{22.1} \approx 1.85.$$

There is no solution.

9b We have

$$\begin{aligned} \frac{\sin B}{10} &= \frac{\sin 10^\circ}{3} \Rightarrow \sin B = 0.57883 \\ &\Rightarrow \sin^{-1}(\sin B) = \sin^{-1}(0.57883) = 35.366^\circ \\ &\Rightarrow \sin B = \sin 35.366^\circ. \end{aligned}$$

One solution to this equation is of course $B_1 = 35.366^\circ$; however B could also be the Quadrant II angle

$$B_2 = 180^\circ - \sin^{-1}(0.57883) = 144.63^\circ$$

(see very pretty picture below).

For the angle B_1 we get $C_1 = 134.634^\circ$, and then by the Law of Cosines we obtain

$$c_1^2 = a^2 + b^2 - 2ab \cos C_1 = 3^2 + 10^2 - 2(3)(10) \cos 134.634^\circ = 151.156 \Rightarrow c_1 = 12.29.$$

So one possible triangle (rounding to the nearest tenth) has

$$B_1 = 35.4^\circ, \quad C_1 = 134.6^\circ, \quad c_1 = 12.3.$$

For the angle B_2 we get $C_2 = 25.370^\circ$, and then by the Law of Cosines we obtain

$$c_2^2 = a^2 + b^2 - 2ab \cos C_2 = 3^2 + 10^2 - 2(3)(10) \cos 25.370^\circ = 54.786 \Rightarrow c_2 = 7.40.$$

So another possible triangle has

$$B_2 = 144.6^\circ, \quad C_2 = 25.4^\circ, \quad c_2 = 7.4.$$

9c The Law of Cosines is necessary here:

$$\begin{aligned}c^2 &= a^2 + b^2 - 2ab \cos C \Rightarrow 6^2 = 4^2 + 3^2 - 2(4)(3) \cos C \\&\Rightarrow \cos C = -11/24 \\&\Rightarrow C = \cos^{-1}(-11/24) \approx 117.28^\circ.\end{aligned}$$

And

$$\begin{aligned}b^2 &= a^2 + c^2 - 2ac \cos B \Rightarrow 3^2 = 4^2 + 6^2 - 2(4)(6) \cos B \\&\Rightarrow \cos B = 43/48 \\&\Rightarrow B = \cos^{-1}(43/48) \approx 26.38^\circ.\end{aligned}$$

Finally, $A = 180^\circ - 26.38^\circ - 117.28^\circ = 36.34^\circ$. To the nearest tenth we have:

$$A = 36.3^\circ, \quad B = 26.4^\circ, \quad C = 117.3^\circ.$$