

MATH 125 EXAM #3 KEY (FALL 2015)

1 For $f(x) = 1 - 2^{x+3}$ we have domain $(-\infty, \infty)$, range $(-\infty, 1)$, and horizontal asymptote $y = 1$.

2 $\text{Dom}(g) = \{x : 2x + 3 > 0\} = \{x : x > -\frac{3}{2}\} = (-\frac{3}{2}, \infty)$.

3a $5^{x^2+8} = 125^{2x}$ becomes $5^{x^2+8} = 5^{6x}$, so that $x^2 + 8 = 6x$ and then

$$x^2 + 8 = 6x \Rightarrow x^2 - 6x + 8 = 0 \Rightarrow (x - 4)(x - 2) = 0 \Rightarrow x = 2, 4.$$

3b We have

$$\log_3(3x - 2) = 2 \Rightarrow 3^2 = 3x - 2 \Rightarrow x = \frac{11}{3}.$$

3c Solving,

$$5e^{x/5} = 7 \Rightarrow e^{x/5} = \frac{7}{5} \Rightarrow \frac{x}{5} = \ln\left(\frac{7}{5}\right) \Rightarrow x = 5 \ln\left(\frac{7}{5}\right).$$

3d Using a law of logarithms,

$$\begin{aligned} \log_5(x + 3) = 1 - \log_5(x - 1) &\Rightarrow \log_5(x + 3) + \log_5(x - 1) = 1 \Rightarrow \log_5(x + 3)(x - 1) = 1 \\ &\Rightarrow (x + 3)(x - 1) = 5 \Rightarrow x^2 + 2x - 8 = 0 \\ &\Rightarrow (x + 4)(x - 2) = 0 \Rightarrow x = -4, 2. \end{aligned}$$

But -4 is an extraneous solution, so the solution set is $\{2\}$.

3e Take the logarithm of both sides (the logarithm base isn't important, though 10 and e are common):

$$\begin{aligned} 2^{x+1} = 5^{1-2x} &\Rightarrow (x + 1) \ln 2 = (1 - 2x) \ln 5 \Rightarrow x \ln 2 + 2x \ln 5 = \ln 5 - \ln 2 \\ &\Rightarrow x = \frac{\ln 5 - \ln 2}{\ln 2 + 2 \ln 5} = \frac{\ln(5/2)}{\ln 50}. \end{aligned}$$

4 By the laws of logarithms,

$$2 \log_8 u - 3 \log_8 v = \log_8(u^2) - \log_8(v^3) = \log_8\left(\frac{u^2}{v^3}\right).$$

5a From $A(t) = 100e^{-0.087t}$ we have $A(7) = 100e^{-0.087(7)} \approx 54.4$ grams.

5b From $A(t) = 5$ we have

$$100e^{-0.087t} = 5 \Rightarrow e^{-0.087t} = 0.05 \Rightarrow -0.087t = \ln(0.05) \Rightarrow t = -\frac{\ln(0.05)}{0.087} \approx 34.4 \text{ days.}$$

5c Given an initial amount A_0 , we find the time t for which $A(t) = \frac{1}{2}A_0$:

$$A_0e^{-0.087t} = \frac{A_0}{2} \Rightarrow e^{-0.087t} = 0.5 \Rightarrow -0.087t = \ln(0.5) \Rightarrow t = -\frac{\ln(0.5)}{0.087} \approx 8.0 \text{ days.}$$

6 $29.411^\circ = 29^\circ 24.66' = 29^\circ 24' 39.6'' \approx 29^\circ 24' 40''.$

7 Angle θ is in Quadrant III. Thus $\tan \theta = \frac{3}{4}$, $\sin \theta = -\frac{3}{5}$, $\cos \theta = -\frac{4}{5}$, $\csc \theta = -\frac{5}{3}$, $\sec \theta = -\frac{5}{4}$.

8 Domain is $(-\infty, \infty)$, range is $[-2, 6]$.

9 Domain is

$$\begin{aligned} \text{Dom}(g) &= \{x : x/4 \neq n\pi \text{ for any } n \in \mathbb{Z}\} = \{x : x \neq 4n\pi \text{ for any } n \in \mathbb{Z}\} \\ &= \cdots \cup (-12\pi, -8\pi) \cup (-8\pi, -4\pi) \cup (0, 4\pi) \cup (4\pi, 8\pi) \cup (8\pi, 12\pi) \cup \cdots, \end{aligned}$$

and range is $(-\infty, \infty)$.

10 Amplitude is 2, period is 1, and phase shift is $-\frac{2}{\pi}$.