Math 125 Exam #2 Key (Fall 2015)

1 $f(x) = (x+2)^2(x-4)^3$.

2a $\dfrac{\text{Factor of } 30}{\text{Factor of } 2} = \pm1, \pm2, \pm3, \pm5, \pm6, \pm10, \pm15, \pm30, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}$.

2b The division

$$\begin{array}{r|rrr|r} 2 & 2 & -3 & -17 & 30 \\ & & 4 & 2 & -30 \\ \hline & 2 & 1 & -15 & 0 \end{array}$$

shows that 2 is a zero for f, and we obtain the factorization

$$f(x) = (x-2)(2x^2 + x - 15) = (x-2)(2x-5)(x+3)$$

So the zeros of f are: 2, $\frac{5}{2}$, -3.

2c $f(x) = (x-2)(2x-5)(x+3)$.

3 Possible rational zeros are $\pm1, \pm5, \pm17, \pm85$. It turns out -5 works:

$$\begin{array}{r|rrr|r} -5 & 1 & 13 & 57 & 85 \\ & & -5 & -40 & -85 \\ \hline & 1 & 8 & 17 & 0 \end{array}$$

Now we have $p(x) = (x+5)(x^2+8x+17)$. Solving $x^2+8x+17 = 0$ with the quadratic formula (or completing the square) yields $x = -4 \pm i$. Thus the zeros of p are -5, $-4-i$, $-4+i$.

4 Factoring, we have

$$r(x) = \frac{(2x+3)(x-4)}{(3x+1)(x-4)}.$$

This makes clear that $x = -\frac{1}{3}$ is the only vertical asymptote. The only horizontal asymptote is $y = \frac{2}{3}$. There is no oblique asymptote.

5a Factoring gives $x(x-1)(x+3) > 0$. There are four cases to consider:

$$x > 0,\ x-1 > 0,\ x+3 > 0; \quad x < 0,\ x-1 < 0,\ x+3 > 0;$$

$$x > 0,\ x-1 < 0,\ x+3 < 0; \quad x < 0,\ x-1 > 0,\ x+3 < 0.$$

The 1st case yields $x > 1$, the 2nd case yields $-3 < x < 0$, and the remaining cases lead to contradictions. Solution set: $(-3,0) \cup (1,\infty)$.

5b We need 0 on one side and a single quotient on the other:

$$\frac{x+2}{x-4} \geq 1 \quad\Leftrightarrow\quad \frac{x+2}{x-4} - 1 \geq 0 \quad\Leftrightarrow\quad \frac{x+2}{x-4} - \frac{x-4}{x-4} \geq 0 \quad\Leftrightarrow\quad \frac{6}{x-4} \geq 0.$$

The last equality is only satisfied if $x - 4 > 0$. Solution set: $(4,\infty)$.

6 We have

$$(f \circ g)(8) = f(g(8)) = f\big(\sqrt[3]{8}\big) = f(2) = \frac{3}{2-1} = 3,$$
$$(g \circ f)(2) = g(f(2)) = g(3) = \sqrt[3]{3},$$
$$(f \circ f)(-3) = f(f(-3)) = f\left(\frac{3}{-3-1}\right) = f\big(-\tfrac{3}{4}\big) = \frac{3}{-\frac{3}{4}-1} = -\tfrac{12}{7},$$
$$(g \circ g)(-64) = g(g(-64)) = g\big(\sqrt[3]{-64}\big) = g(-4) = \sqrt[3]{-4} = -\sqrt[3]{4}.$$

7a We have $(f \circ g)(x) = f(g(x)) = f(\sqrt{x-2}) = (\sqrt{x-2})^2 + 4 = (x-2) + 4 = x + 2$, with domain $[2, \infty)$.

7b We have $(g \circ f)(x) = g(f(x)) = f(x^2 + 4) = \sqrt{x^2 + 4}$, with domain $(-\infty, \infty)$.

7c We have $(g \circ g)(x) = g(g(x)) = g(\sqrt{x-2}) = \sqrt{\sqrt{x-2} - 2}$, with domain

$$\{x : x \in \mathrm{Dom}(g) \quad \text{and} \quad g(x) \in \mathrm{Dom}(g)\},$$

where $\mathrm{Dom}(g) = [2, \infty)$. Hence we must have $x \geq 2$ and $g(x) \geq 2$. Since

$$g(x) \geq 2 \quad \Rightarrow \quad \sqrt{x-2} \geq 2 \quad \Rightarrow \quad x - 2 \geq 4 \quad \Rightarrow \quad x \geq 6,$$

it follows that $\mathrm{Dom}(g \circ g) = [6, \infty)$.

8a $f^{-1}(x) = \sqrt[3]{x-9}$.

8b $f^{-1}(x) = \dfrac{x}{x+2}$.