

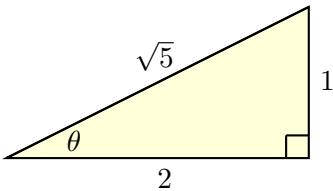
MATH 125 EXAM #4 KEY (FALL 2010)

1. Let $\theta = \sin^{-1}(\sin 9\pi/8)$. Then θ is a value in $[-\pi/2, \pi/2]$ such that $\sin \theta = \sin(9\pi/8)$, and thus $\theta = -\pi/8$. That is, $\sin^{-1}(\sin 9\pi/8) = -\pi/8$.

2. We restrict f to $[-\pi/2, \pi/2]$ to obtain a function that is one-to-one and therefore has an inverse, so $\text{Dom}(f) = [-\pi/2, \pi/2] = \text{Ran}(f^{-1})$. As for the range of f , $-1 \leq \sin x \leq 1 \Rightarrow -5 \leq 5 \sin x \leq 5 \Rightarrow -8 \leq 5 \sin x - 3 \leq 2$, so $\text{Ran}(f) = [-8, 2] = \text{Dom}(f^{-1})$. Finally, let $y = f(x)$, so $y = 5 \sin x - 3$ and we get $x = \sin^{-1}\left(\frac{y+3}{5}\right)$. Since $y = f(x)$ implies that $x = f^{-1}(y)$, the result is $f^{-1}(y) = \sin^{-1}\left(\frac{y+3}{5}\right)$.

3a. Let $\theta = \sin^{-1}(-1/2)$, so $\sin \theta = -1/2$ for $\theta \in [-\pi/2, \pi/2]$, which implies that $\theta = -\pi/6$. Now, $\tan(\sin^{-1}(-1/2)) = \tan(-\pi/6) = -1/\sqrt{3}$.

3b. Let $\theta = \tan^{-1}(1/2)$, so $\tan \theta = 1/2$ for $\theta \in (-\pi/2, \pi/2)$, which implies that θ is the angle in the right triangle shown below. Then $\sec(\tan^{-1}(1/2)) = \sec \theta = \sqrt{5}/2$.



4. $\theta = \csc^{-1}(5) \Rightarrow \csc \theta = 5 \Rightarrow \sin \theta = 1/5$ for $\theta \in [-\pi/2, 0) \cup (0, \pi/2]$ (which is the range of the \csc^{-1} function). Using a calculator, we find that $\theta = \sin^{-1}(1/5) \approx 0.2014 \approx 0.20$.

5a.
$$\frac{\sec \theta}{\csc \theta} + \frac{\sin \theta}{\cos \theta} = \frac{\sin \theta}{\cos \theta} + \frac{\sin \theta}{\cos \theta} = 2 \cdot \frac{\sin \theta}{\cos \theta} = 2 \tan \theta.$$

5b.
$$3 \sin^2 \theta + 4 \cos^2 \theta = 3(1 - \cos^2 \theta) + 4 \cos^2 \theta = 3 - 3 \cos^2 \theta + 4 \cos^2 \theta = 3 + \cos^2 \theta.$$

5c.
$$\cos\left(\frac{3\pi}{2} + \theta\right) = \cos(3\pi/2) \cos \theta - \sin(3\pi/2) \sin \theta = 0 \cdot \cos \theta - (-1) \sin \theta = \sin \theta.$$

5d.
$$\cos^4 \theta - \sin^4 \theta = (\cos^2 \theta - \sin^2 \theta)(\cos^2 \theta + \sin^2 \theta) = \cos(2\theta) \cdot 1 = \cos(2\theta).$$

6a. The angle $19\pi/12$ is in Quadrant IV where tangent is negative, so $\tan\left(\frac{19\pi}{12}\right) = -\sqrt{\frac{1 - \cos(19\pi/6)}{1 + \cos(19\pi/6)}} = -\sqrt{\frac{1 - (-\sqrt{3}/2)}{1 + (-\sqrt{3}/2)}} = -\sqrt{\frac{2 + \sqrt{3}}{2 - \sqrt{3}}} = -2 - \sqrt{3}$, where $\cos(19\pi/6) = \cos(7\pi/6)$.

6b.
$$\cos 40^\circ \cos 10^\circ + \sin 40^\circ \sin 10^\circ = \cos(40^\circ - 10^\circ) = \cos 30^\circ = \sqrt{3}/2.$$

6c. The angle 165° is in Quadrant II where cosine is negative, so $\cos 165^\circ = -\sqrt{\frac{1 + \cos 330^\circ}{2}} = -\sqrt{\frac{1 + \sqrt{3}/2}{2}}$

$$= -\sqrt{\frac{2+\sqrt{3}}{4}} = -\frac{\sqrt{6}+\sqrt{2}}{4}, \text{ where the last equality holds since } \left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)^2 = \frac{6+2\sqrt{12}+2}{16} = \frac{8+4\sqrt{3}}{16}$$

$$= \frac{2+\sqrt{3}}{4}.$$

7a. $3\sqrt{2}\cos\theta = -3 \Rightarrow \cos\theta = -1/\sqrt{2} \Rightarrow \theta = 3\pi/4, 5\pi/4.$

7b. $2\sin^2\theta = 3(1-\cos\theta) \Rightarrow 2(1-\cos^2\theta) = 3-3\cos\theta \Rightarrow 2\cos^2\theta-3\cos\theta+1=0 \Rightarrow (2\cos\theta-1)(\cos\theta-1)=0,$
so either $\cos\theta = 1/2$ or $\cos\theta = 1$. Solving these equations gives $\theta = \pi/3, 5\pi/3, 0.$

7c. $\cos(2\theta) = 2-2\sin^2\theta \Rightarrow \cos^2\theta - \sin^2\theta = 2-2\sin^2\theta \Rightarrow (1-\sin^2\theta) - \sin^2\theta = 2-2\sin^2\theta \Rightarrow 1-2\sin^2\theta = 2-2\sin^2\theta \Rightarrow 1=2.$ No solution.

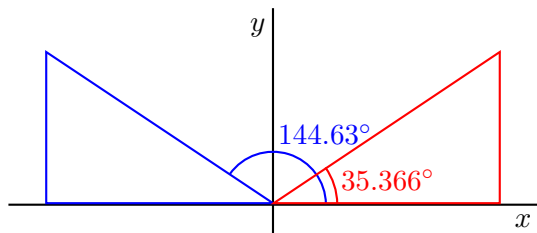
8. First, $C = 180^\circ - 40^\circ - 45^\circ = 95^\circ.$ Now, by Ye Olde Law of Sines, $\frac{a}{\sin 45^\circ} = \frac{4}{\sin 95^\circ} \Rightarrow a = 2.839,$ and $\frac{b}{\sin 40^\circ} = \frac{4}{\sin 95^\circ} \Rightarrow b = 2.581.$

9a. First, $A = 180^\circ - 10^\circ - 100^\circ = 70^\circ.$ Now, $\frac{c}{\sin 100^\circ} = \frac{2}{\sin 10^\circ} \Rightarrow c = 11.343,$ and $\frac{a}{\sin 70^\circ} = \frac{2}{\sin 10^\circ} \Rightarrow a = 10.823.$

9b. $\frac{\sin B}{10} = \frac{\sin 10^\circ}{3} \Rightarrow \sin B = 0.57883 \Rightarrow \sin^{-1}(\sin B) = \sin^{-1}(0.57883) = 35.366^\circ \Rightarrow \sin B = \sin 35.366^\circ.$
One solution to this equation is of course $B_1 = 35.366^\circ;$ however B could also be the Quadrant II angle $B_2 = 180^\circ - \sin^{-1}(0.57883) = 144.63^\circ$ (see very pretty picture below).

For the angle B_1 we get $C_1 = 134.634^\circ,$ and then by the Law of Cosines we obtain $c_1^2 = a^2 + b^2 - 2ab\cos C_1 = 3^2 + 10^2 - 2(3)(10)\cos 134.634^\circ = 151.156 \Rightarrow c_1 = 12.29.$ So one possible triangle (rounding to the nearest hundredth) has $B_1 = 35.37^\circ, C_1 = 134.63^\circ, c_1 = 12.29.$

For the angle B_2 we get $C_2 = 25.370^\circ,$ and then by the Law of Cosines we obtain $c_2^2 = a^2 + b^2 - 2ab\cos C_2 = 3^2 + 10^2 - 2(3)(10)\cos 25.370^\circ = 54.786 \Rightarrow c_2 = 7.40.$ So another possible triangle has $B_2 = 144.63^\circ, C_2 = 25.37^\circ, c_2 = 7.40.$



9c. The Law of Cosines is necessary here: $c^2 = a^2 + b^2 - 2ab\cos C \Rightarrow 6^2 = 4^2 + 3^2 - 2(4)(3)\cos C \Rightarrow \cos C = -11/24 \Rightarrow C = \cos^{-1}(-11/24) = 117.28^\circ.$ And $b^2 = a^2 + c^2 - 2ac\cos B \Rightarrow 3^2 = 4^2 + 6^2 - 2(4)(6)\cos B \Rightarrow \cos B = 43/48 \Rightarrow B = \cos^{-1}(43/48) = 26.38^\circ.$ Finally, $A = 180^\circ - 26.38^\circ - 117.28^\circ = 36.34^\circ.$