Math 125 Exam #3 Key (Fall 2010)

1. $(3^2)^{-x+15} = (3^3)^x \;\Rightarrow\; 3^{-2x+30} = 3^{3x} \;\Rightarrow\; -2x+30 = 3x \;\Rightarrow\; x = 6$

2. $\text{Dom}\, f = \{x \,|\, 2x-3 > 0\} = \{x \,|\, x > 3/2\} = \left(\frac{3}{2}, \infty\right)$. Now, let $y = f(x)$, so $y = 8 + 5\log_3(2x-3) \;\Rightarrow\; \dfrac{y-8}{5} = \log_3(2x-3) \;\Rightarrow\; 3^{(y-8)/5} = 2x-3 \;\Rightarrow\; x = \frac{1}{2}\left(3^{\frac{y-8}{5}} + 3\right) \;\Rightarrow\; f^{-1}(y) = \frac{1}{2}\left(3^{\frac{y-8}{5}} + 3\right)$, or equivalently $f^{-1}(x) = \frac{1}{2}\left(3^{\frac{x-8}{5}} + 3\right)$.

3a. $x = 5^3 = 125$.

3b. $x^2 + 1 = 3^2 \;\Rightarrow\; x^2 = 8 \;\Rightarrow\; x = \pm 2\sqrt{2}$.

4. $\log_7\left(\dfrac{u^2}{v}\right)$.

5. $\ln(y-3) = \ln e^{-4x} + \ln C = \ln(Ce^{-4x}) \;\Rightarrow\; y - 3 = Ce^{-4x} \;\Rightarrow\; y = 3 + Ce^{-4x}$.

6a. $2x + 3 = 3 \;\Rightarrow\; x = 0$.

6b. $\log\left(\dfrac{2x}{x-3}\right) = 1 \;\Rightarrow\; \dfrac{2x}{x-3} = 10^1 \;\Rightarrow\; 2x = 10(x-3) \;\Rightarrow\; x = \frac{15}{4}$.

6c. $\ln 3^x = \ln 14 \;\Rightarrow\; x \ln 3 = \ln 14 \;\Rightarrow\; x = \dfrac{\ln 14}{\ln 3} \approx 2.402$.

7. $29.411^\circ = 29^\circ + (0.411^\circ)\left(\dfrac{60'}{1^\circ}\right) = 29^\circ + 24.66' = 29^\circ + 24' + (0.66')\left(\dfrac{60''}{1'}\right) = 29^\circ + 24' + 39.6'' \approx 29^\circ 24' 40''$

8a. $\dfrac{11}{6}\pi$ **8b.** -150°

9. $\sin\theta = -\frac{12}{13}$, $\cos\theta = \frac{5}{13}$, $\tan\theta = -\frac{12}{5}$, $\sec\theta = \frac{13}{5}$, $\csc\theta = -\frac{13}{12}$, $\cot\theta = -\frac{5}{12}$.

10. The angle 540° has the same terminal side as 180°, so $\sec 540^\circ = \sec 180^\circ = 1/\cos 180^\circ = 1/-1 = -1$.

11. We must be in Quadrant II with $x = -3$, $y = 4$ and $r = 5$. Hence $\tan\theta = -\frac{4}{3}$, $\cot\theta = -\frac{3}{4}$, $\sec\theta = -\frac{5}{3}$, $\csc\theta = \frac{5}{4}$.

12. We must have $x = -4$, $y = -3$ and $r = 5$. Hence $\sin\theta = -\frac{3}{5}$, $\tan\theta = \frac{3}{4}$, $\cot\theta = \frac{4}{3}$, $\sec\theta = -\frac{5}{4}$, $\csc\theta = -\frac{5}{3}$.

13. Period is $\dfrac{2\pi}{3/2} = \dfrac{4}{3}\pi$ and amplitude is $\left|-\dfrac{1}{2}\right| = \dfrac{1}{2}$.