MATH 125 EXAM #2 KEY (FALL 2010)

1. $f^{-1}(x) = \sqrt[3]{x-7}$

2. $y = \dfrac{2x-3}{x+4} \Rightarrow xy + 4y = 2x - 3 \Rightarrow xy - 2x = -4y - 3 \Rightarrow x(y-2) = -4y - 3 \Rightarrow x = -\dfrac{4y+3}{y-2}$, and so $h^{-1}(x) = -\dfrac{4x+3}{x-2}$. $\operatorname{Dom} h = \operatorname{Ran} h^{-1} = (-\infty, -4) \cup (-4, \infty)$ and $\operatorname{Ran} h = \operatorname{Dom} h^{-1} = (-\infty, 2) \cup (2, \infty)$

3. Not linear since average rate of change is not constant: from $(0,0)$ to $(10,3)$ average is $3/10 = 0.3$, but from $(10,3)$ to $(20,8)$ average is $5/10 = 0.5$

4a. Vertex is at $\left(\frac{1}{4}, \frac{15}{8}\right)$, axis is $x = \frac{1}{4}$.

4b. Vertex form: $f(x) = 2\left(x - \frac{1}{4}\right)^2 + \frac{15}{8}$. Domain: $(-\infty, \infty)$. Range: $\left(\frac{15}{8}, \infty\right)$.

5. An arbitrary point on the line has coordinates (x,x), and distance between (x,x) and $(-1,2)$ is given by $d(x) = \sqrt{(x+1)^2 + (x-2)^2}$, or equivalently $d^2(x) = 2x^2 - 2x + 5$. A little algebra gives $d^2(x) = 2\left(x - \frac{1}{2}\right)^2 + \frac{9}{2}$, which hits its minimum value when $x = \frac{1}{2}$. Hence $\left(\frac{1}{2}, \frac{1}{2}\right)$ is the point on $y = x$ that's closest to $(-1,2)$.

6a. $x(x+1) > 20 \Rightarrow (x+5)(x-4) > 0$. Case I: $x + 5 > 0$ & $x - 4 > 0$, which gives $x > 4$. Case II: $x + 5 < 0$ & $x - 4 < 0$, which gives $x < -5$. Solution set: $(-\infty, -5) \cup (4, \infty)$.

6b. $x^3 - 2x^2 - 3x < 0 \Rightarrow x(x-3)(x+1) < 0$. Case I: $x < 0$ & $x - 3 < 0$ & $x + 1 < 0$, giving $x < -1$. Case II: $x < 0$ & $x - 3 > 0$ & $x + 1 > 0$, giving a contradiction. Case III: $x > 0$ & $x - 3 < 0$ & $x + 1 > 0$, giving $0 < x < 3$. Case IV: $x > 0$ & $x - 3 > 0$ & $x + 1 < 0$, giving a contradiction. Solution set: $(-\infty, -1) \cup (0, 3)$.

6c. $\dfrac{3x-5}{x+2} - 2 \leq 0 \Rightarrow \dfrac{3x-5}{x+2} - \dfrac{2(x+2)}{x+2} \leq 0 \Rightarrow \dfrac{x-9}{x+2} \leq 0$. Case I: $x - 9 \leq 0$ & $x + 2 > 0$, giving $-2 < x \leq 9$. Case II: $x - 9 \geq 0$ & $x + 2 < 0$, giving a contradiction. Solution set: $(-2, 9]$.

7. $f(x) = (x+2)(x-1)(x-4)$, or $f(x) = x^3 - 3x^2 - 6x + 8$. Extra Credit: Since f as constructed so far gives $f(5) = 28$, in order to have $f(5) = 16$ we must put in a factor of $\frac{16}{28}$ to get: $f(x) = \frac{4}{7}(x^3 + 5x^2 + 2x - 8)$.

8. $R(x) = \dfrac{(x-2)(x^2+2x+4)}{(x-3)(x-2)} = \dfrac{x^2+2x+4}{x-3}$, so $x = 3$ is a vertical asymptote for R. Long division gives $R(x) = (x+5) + \dfrac{19}{x-3}$, which shows that $y = x + 5$ is an oblique asymptote for R.

9.

1	2	11	−5	−43	35
		2	13	8	−35
	2	13	8	−35	0

$\longrightarrow \; f(x) = (x-1)(2x^3 + 13x^2 + 8x - 35)$

−5	2	13	8	−35
		−10	−15	35
	2	3	−7	0

$\longrightarrow \; f(x) = (x-1)(x+5)(2x^2 + 3x - 7)$

Complete list of zeros: -5, 1, $\dfrac{-3 \pm \sqrt{65}}{4}$ (use quadratic formula to get last two).

Complete factorization: $f(x) = (x-1)(x+5)\left(x + \dfrac{3+\sqrt{65}}{4}\right)\left(x + \dfrac{3-\sqrt{65}}{4}\right)$

10. To have real coefficients we must have $3-2i$ also be a zero. Then $f(x) = (x-4)^2[x-(3+2i)][x-(3-2i)]$, which in standard form is $f(x) = x^4 - 14x^3 + 77x^2 - 200x + 208$.

11.

$$\begin{array}{r|rrr|r} 2 & 1 & -8 & 25 & -26 \\ & & 2 & -12 & 26 \\ \hline & 1 & -6 & 13 & 0 \end{array} \quad \longrightarrow \quad \text{So 2 is a zero and } f(x) = (x-2)(x^2-6x+13)$$

Applying ye olde quadratic formula to $x^2 - 6x + 13 = 0$ yields the zeros $3 \pm 2i$. In conclusion, f has zeros 2, $3-2i$, $3+2i$, and $f(x)$ has factorization $(x-2)(x-3+2i)(x-3-2i)$.