Math 125 Exam #1 Key (Fall 2010)

1. $D = \sqrt{(10-2)^2 + (3-(-3))^2} = \sqrt{100} = 10.$

2. y-intercept: set $x = 0$ and solve for y to get $y = 9$; so $(0,9)$ is the y-intercept. x-intercept: set $y = 0$ to get $9x^2 = 36 \;\Rightarrow\; x = \pm 2$; so $(\pm 2, 0)$ are the x-intercepts. Symmetry is with respect to the y-axis.

3. $4(x-4) - 5 \cdot 4(x+1) = 21(x+1)(x-4) \;\Rightarrow\; 21x^2 - 47x - 48 = 0 \;\Rightarrow\; x = \dfrac{47 \pm \sqrt{47^2 - 4(21)(-48)}}{42} = \dfrac{47 \pm 79}{42} \in \left\{3,\ -\dfrac{16}{21}\right\}.$

4. Slope of line is $m = \dfrac{-8-(-5)}{-6-2} = \dfrac{3}{8}$. Now use the point-slope formula to get $y + 5 = \dfrac{3}{8}(x-2)$, which leads to $y = \dfrac{3}{8}x - \dfrac{23}{4}$.

5. The center of the circle lies at the midpoint between $(1,4)$ & $(-3,2)$, at $\left(\dfrac{1+(-3)}{2}, \dfrac{4+2}{2}\right) = (-1,3)$. The radius of the circle is the distance between $(-1,3)$ and $(1,4)$: $r = \sqrt{2^2 + 1^2} = \sqrt{5}$. Equation of circle: $(x+1)^2 + (y-3)^2 = 5$.

6. $f(-1) = -\frac{1}{2}$ and $f(x+1) = \dfrac{x+1}{(x+1)^2 + 1}$.

7a. $\text{Dom } f = \{x \mid x \neq \pm 6\}$

7b. $\text{Dom } g = \{x \mid 3x \geq 12\} = \{x \mid x \geq 4\} = [4, \infty)$.

7c. $\text{Dom } f + g = \text{Dom } f \cap \text{Dom } g = [4,6) \cup (6, \infty)$.

7d. $\text{Dom } f/g = (4,6) \cup (6,\infty)$, where we have to exclude 4 from $\text{Dom } f \cap \text{Dom } g$ since $g(4) = 0$.

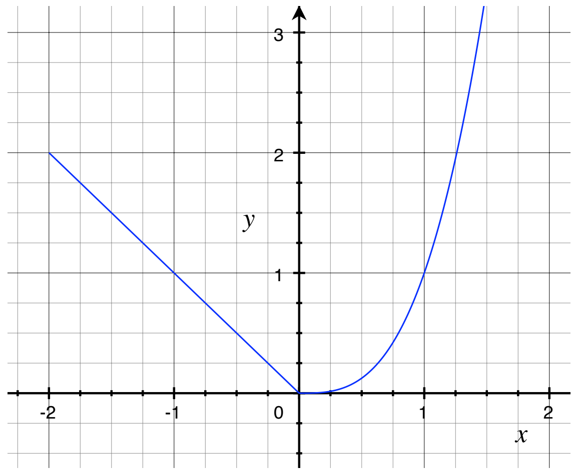
8a. Yes, since $f(-1) = 2(-1)^2 - (-1) - 1 = 2$.

8b. $f(-2) = 2(-2)^2 - (-2) - 1 = 9$, and the corresponding point on the graph is $(-2, 9)$.

8c. We have $2x^2 - x - 1 = -1$, whence $2x^2 - x = 0 \;\Rightarrow\; x(2x-1) = 0 \;\Rightarrow\; x = 0, \frac{1}{2}$, and the corresponding points on the graph are $(0,-1)$ and $(1/2, -1)$.

9a. Dom $f = [-2, 0) \cup (0, \infty)$ and Ran $f = (0, \infty)$.

9b. Note: there will be an open circle at the origin, but it was going to be WWIII with the graphing application I was using to try to get it in there.



10. Start with $f(x) = \sqrt{x}$. Shift up 3 units gives a new function: $g(x) = f(x)+3$. Reflecting g about the x-axis gives a new function: $h(x) = -g(x)$. Shifting left by 5 units gives yet another new function: $k(x) = h(x+5)$. Our result is: $k(x) = h(x+5) = -g(x+5) = -[f(x+5)+3]$, or $y = -(\sqrt{x+5}+3) = -\sqrt{x+5}-3$.

11. In general $A = \pi r^2$. But circumference C is given by $C = 2\pi r$ and we know that C must equal x. Hence $x = 2\pi r$, which yields $r = \dfrac{x}{2\pi}$. Therefore A as a function of x is given by: $A(x) = \pi\left(\dfrac{x}{2\pi}\right)^2$, or $A(x) = \dfrac{x^2}{4\pi}$.

12. $(f \circ g)(4) = f(g(4)) = f(12) = \sqrt{13}$ and $(g \circ f)(2) = g(f(2)) = g(\sqrt{3}) = 3\sqrt{3}$.

13a $$(f \circ f)(x) = f(f(x)) = \frac{f(x)-5}{f(x)+1} = \frac{\dfrac{x-5}{x+1}-5}{\dfrac{x-5}{x+1}+1} = \frac{2x+5}{2-x}.$$

13b. First, Dom $f = \{x \mid x \neq -1\}$ and Dom $g = \{x \mid x \neq 3\}$. Now, by definition, Dom $g \circ f = \{x \mid x \in \text{Dom } f \text{ and } f(x) \in \text{Dom } g\} = \left\{x \,\middle|\, x \neq -1 \text{ and } \dfrac{x-5}{x+1} \neq 3\right\} = \{x \mid x \neq -1 \text{ and } x \neq -4\}$.

13c. By definition, Dom $f \circ g = \{x \mid x \in \text{Dom } g \text{ and } g(x) \in \text{Dom } f\} = \left\{x \,\middle|\, x \neq 3 \text{ and } \dfrac{x+2}{x-3} \neq -1\right\} = \{x \mid x \neq 3 \text{ and } x \neq 1/2\}$.