1. [10 pts.] Given that $\csc\theta = 3$ and $\cot\theta < 0$, find the value of all trigonometric functions of θ.

2. [10 pts.] If $f(\theta) = \tan\theta$ and $f(a) = 6$, find the exact value of $f(-a)$ and $f(a + \pi) + f(a - 3\pi)$.

3. [10 pts. each] Find the exact value of each expression, or state that it is undefined.

   (a) $\cos^{-1}\left(\cos\frac{3\pi}{10}\right)$

   (b) $\sin^{-1}\left(\sin\frac{6\pi}{7}\right)$

   (c) $\csc\left(\tan^{-1}\frac{1}{2}\right)$

   (d) $\sec^{-1}\left(\sin\frac{7\pi}{6}\right)$

4. [20 pts.] Find the inverse function f^{-1} for $f(x) = -2\cos(3x)$, $0 \le x \le \pi/3$. Using interval notation, find the range of f and the domain and range of f^{-1}.

5. [10 pts. each] Solve each trigonometric equation on the interval $0 \le \theta < 2\pi$.

   (a) $2\cos^2\theta - 1 = 0$

   (b) $\tan\theta + 1 = 0$

   (c) $4(1 + \sin\theta) = \cos^2\theta$

6. [10 pts. each] Establish the identity.

   (a) $1 - \dfrac{\sin^2\theta}{1 - \cos\theta} = -\cos\theta$

   (b) $\sec\theta - \cos\theta = \sin\theta\tan\theta$

7. [10 pts.] Find the exact value of $\tan 15°$ using a double-angle or half-angle formula, showing all work.

8. [10 pts.] A security camera in a neighborhood bank is mounted on a wall 9 feet above the floor. What angle of depression should be used if the camera is to be directed to a spot 6 feet above the floor and 16 feet from the wall?

9. [10 pts.] The angle of elevation of the sun is $35.1°$ at the instant the shadow cast by the Washington Monument is 789 feet long. Use this information to find the height of the monument to the nearest tenth of a foot.

10. [10 pts. each] Solve the triangle using the Law of Sines, rounding sides to the hundredths place and angles to the tenths place when necessary. If no triangle results from the given information, then explain why.

    (a) $A = 50°$, $C = 20°$, $a = 3$.

    (b) $a = 2$, $c = 1$, $C = 100°$.

11. [20 pts.] The distance from the Sun to Earth is about 149,600,000 km, and the distance from the Sun to Venus is about 108,200,000 km. The elongation angle α is the angle formed between the line of sight from Earth to the Sun and the line of sight from Earth to Venus. Assuming that $\alpha = 40°$, find the possible distances between Earth and Venus. Round to the nearest 100,000 km.
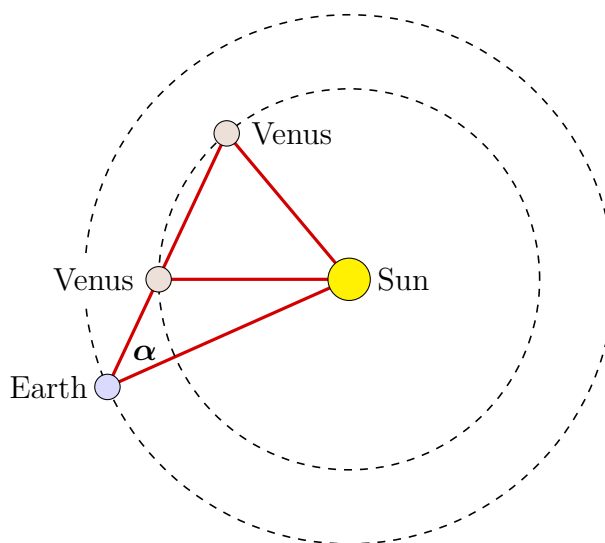
TRIGONOMETRIC IDENTITIES

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

$$\sin 2u = 2 \sin u \cos u$$

$$\cos 2u = \cos^2 u - \sin^2 u$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$

$$\sin \frac{u}{2} = \pm \sqrt{\frac{1 - \cos u}{2}}$$

$$\cos \frac{u}{2} = \pm \sqrt{\frac{1 + \cos u}{2}}$$

$$\tan \frac{u}{2} = \frac{\sin u}{1 + \cos u}$$