MATH 125
FALL 2021 NAME:
EXAM 3

1. [10 pts. each] Let $f(x) = \dfrac{x}{x-4}$ and $g(x) = \dfrac{2}{\sqrt{x}}$.
   (a) Find $(f \circ g)(x)$ and $(g \circ f)(x)$. Do not simplify.
   (b) Find the domain of $f \circ g$, using the appropriate definition.
   (c) Find the domain of $g \circ f$, using the appropriate definition.

2. [10 pts.] If $f(x) = 3x^2 - 7$ and $g(x) = 2x + c$, find the value of c so that the graph of $f \circ g$ crosses the y-axis at 68.

3. [10 pts. each] Let $T(x) = \dfrac{2x-3}{x+4}$, which is one-to-one.
   (a) Find T^{-1}.
   (b) Find the domain and range of T and T^{-1}.

4. [10 pts.] Find the inverse of the function $s(x) = \sqrt{r^2 - x^2}$, where $r > 0$ is a constant and $0 \le x \le r$.

5. [10 pts. each] Solve each equation in exact form. The change-of-base formula may be necessary.
   (a) $5^{x+3} = \frac{1}{25}$
   (b) $\log_x(\frac{1}{8}) = 3$
   (c) $2\log_3(x+4) - \log_3 9 = 2$
   (d) $\log_5(x+3) = 1 - \log_5(x-1)$
   (e) $9^x - 3^{x+1} + 1 = 0$
   (f) $\log_2 x + \log_4 x + \log_8 x = 11$

6. [10 pts. each] Let $v(x) = 8 - \log_6(2x+1)$, which is one-to-one.
   (a) Find v^{-1}.
   (b) Find the domain and range of v and v^{-1}.

7. [5 pts. each] The number N of bacteria present in a culture at time t (in hours) obeys the model $N(t) = 1000e^{0.01t}$. Label your answers with the correct units. Round answers to the nearest tenth (one decimal place). Show your work.
   (a) Determine the initial number of bacteria.
   (b) What is the population after 4 hours?
   (c) When will the number of bacteria reach 1600?

8. [15 pts.] Salt (NaCl) decomposes in water into sodium (Na^+) and chloride (Cl^-) ions according to the law of uninhibited decay. If the initial amount of salt is 25 kg and, after 10 hours, 15 kg of salt is left, how much salt will be left after 1 day? How long will it take until 0.3 kg of salt is left?

9. [10 pts.] Convert 23.907° to degree-minute-second format, rounding to the nearest second. Show work.

10. [10 pts.] The terminal side of the angle θ contains the point $(-2, 5)$. Find the exact value of each of the six trigonometric functions of θ.