

1. 5 pts. each Find the exact value of each expression, or state that it is undefined.

(a)  $\tan^{-1}\left[\tan\left(-\frac{2\pi}{3}\right)\right]$

(b)  $\cos^{-1}\left[\sin\left(\frac{7\pi}{6}\right)\right]$

2. 10 pts. each Let  $f(x) = 2 \tan x - 3$  for  $-\frac{\pi}{2} < x < \frac{\pi}{2}$ .

(a) Find the inverse function  $f^{-1}$ .

(b) Find the range of  $f$ , and the domain and range of  $f^{-1}$ .

3. 10 pts. each Solve each trigonometric equation on the interval  $0 \leq \theta < 2\pi$ .

(a)  $2 \cos^2 \theta + \cos \theta - 1 = 0$

(b)  $(\cot \theta + 1)\left(\csc \theta - \frac{1}{2}\right) = 0$

4. 10 pts. each Establish the identity, showing all steps.

(a)  $\frac{\cos v}{1 + \sin v} + \frac{1 + \sin v}{\cos v} = 2 \sec v$

(b)  $\tan \theta + \cot \theta = \sec \theta \csc \theta$

5. 10 pts. Find the exact value of  $\sin(\alpha + \beta)$  and  $\tan(\alpha - \beta)$  given that

$$\cos \alpha = \frac{1}{2}, \quad -\frac{\pi}{2} < \alpha < 0; \quad \sin \beta = \frac{1}{3}, \quad 0 < \beta < \frac{\pi}{2}$$

6. 10 pts. each Establish the identity

(a)  $\sec(\alpha - \beta) = \frac{\sec \alpha \sec \beta}{1 + \tan \alpha \tan \beta}$

(b)  $\tan(v/2) = \csc v - \cot v$

7. 10 pts. Solve the equation  $\sin(2\theta) = \cos \theta$  on the interval  $0 \leq \theta < 2\pi$ .

8. 10 pts. A 22-foot ladder leaning against a building makes a  $70^\circ$  angle with the ground. How far up the building does the ladder touch?

9. 10 pts. each Solve the triangle, rounding to the tenths place.
- (a)  $A = 50^\circ$ ,  $B = 30^\circ$ ,  $a = 1$ .
  - (b)  $a = 3$ ,  $b = 4$ ,  $A = 10^\circ$ .
  - (c)  $a = 10$ ,  $b = 7$ ,  $c = 8$ .
10. 15 pts. Two observers simultaneously measure the angle of elevation of a helicopter. One angle is measured as  $25^\circ$ , the other as  $40^\circ$ . If the observers are 100 meters apart and the helicopter lies over the line joining them, how high is the helicopter?

### TRIGONOMETRIC IDENTITIES

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

$$\sin 2u = 2 \sin u \cos u$$

$$\cos 2u = \cos^2 u - \sin^2 u$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$

$$\sin \frac{u}{2} = \pm \sqrt{\frac{1 - \cos u}{2}}, \quad \cos \frac{u}{2} = \pm \sqrt{\frac{1 + \cos u}{2}}, \quad \tan \frac{u}{2} = \frac{\sin u}{1 + \cos u}$$