MATH 122
SPRING 2024 NAME:
EXAM 3

1. [10 pts.] Given that $\sin\theta = -\frac{4}{5}$ and $\cos\theta < 0$, find the remaining five trigonometric functions of θ.

2. [10 pts.] Write the expression
$$\frac{\sec^2\theta - 1}{\csc^2\theta - 1}$$
in terms of sine and cosine, and then simplify so that no quotients appear.

3. [10 pts.] Verify the identity step-by-step using the fundamental identities:
$$\frac{\sin^2\theta}{\cos\theta} = \sec\theta - \cos\theta.$$

4. [10 pts.] Find $\cos(s+t)$, given that $\sin s = \frac{2}{3}$, $\sin t = -\frac{1}{3}$, s is in quadrant II, and t is in quadrant IV.

5. [10 pts.] Find the exact value of $\tan 15^\circ$ using an appropriate identity.

6. [10 pts.] Verify the identity:
$$\frac{\cos(\alpha-\beta)}{\cos\alpha\sin\beta} = \tan\alpha + \cot\beta.$$

7. [10 pts.] Find the values of $\sin 2\theta$ and $\cos 2\theta$, given that $\sin\theta = -\frac{\sqrt{5}}{7}$ and $\cos\theta > 0$.

8. [10 pts.] Write $\cos 3x$ in terms of a trigonometric function of x.

9. [10 pts.] Verify the identity:
$$\cot^2\frac{u}{2} = \frac{(1+\cos u)^2}{\sin^2 u}.$$

10. [10 pts.] Find the exact value of $\cos(\tan^{-1}(-2))$ without using a calculator (so show the work).

11. [10 pts.] Find the exact solutions to $2\cos^2\theta - \sqrt{3}\cos\theta = 0$ over the interval $[0, 2\pi)$.

12. [10 pts.] Find the exact solutions to $\sin\theta = \sin 2\theta$ over the interval $[0, 2\pi)$.

Trigonometric Identities

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}.$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta.$$

$$\sin 2\theta = 2\sin\theta\cos\theta.$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}.$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}.$$

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}.$$

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \frac{\sin\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\sin\theta}.$$

$$\cos(90^\circ - \theta) = \sin\theta$$

$$\sin(90^\circ - \theta) = \cos\theta$$

$$\tan(90^\circ - \theta) = \cot\theta$$