Math 122 Exam #4 Key (Spring 2024)

1 $\frac{1}{2} - \frac{\pi}{4} < x < \frac{1}{2} + \frac{\pi}{4}$ implies $2x - 1 \in (-\frac{\pi}{2}, \frac{\pi}{2})$, so that

$$y = \tan(2x-1) \quad \Rightarrow \quad 2x - 1 = \arctan y \quad \Rightarrow \quad x = \frac{1 + \arctan y}{2}.$$

2 Place the transmitter at C. The triangle ABC has interior angles $A = 90° - 52.7° = 37.3°$, $B = 21.3°$, and $C = 180° - A - B = 121.4°$. Also $c = 7.82$, and the distance between A and C is b. By the Law of Sines,

$$b = \frac{7.82 \sin 21.3°}{\sin 121.4°} = 3.33 \text{ km}.$$

3 Two triangles are possible: $B_1 = 74.6°$, $C_1 = 43.7°$, $c_1 = 61.9$ m; and $B_2 = 105.4°$, $C_2 = 12.9°$, $c_2 = 20.0$ m.

4 Let θ be the angle. By the Law of Cosines,

$$57.8^2 = 25.9^2 + 32.5^2 - 2(25.9)(32.5)\cos\theta \quad \Rightarrow \quad \cos\theta = -0.95859 \quad \Rightarrow \quad \theta = 163.45° \approx 163°.$$

5 $6 \operatorname{cis} 135° = 6\big(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\big) = -3\sqrt{2} + 3i\sqrt{2}$.

6 $x + yi = \sqrt{3} - i$ implies $x = \sqrt{3}$ and $y = -1$, so (x, y) is in QIV. Now, $\tan\theta = \frac{y}{x} = -\frac{1}{\sqrt{3}}$ gives $\theta = 330°$. Also $r = \sqrt{x^2 + y^2} = 2$. Answer: $2 \operatorname{cis} 330°$.

7 $r = \sqrt{2^2 + 7^2} = \sqrt{53}$ and $\theta = \arctan\frac{7}{2} = 74.05°$. Answer: $\sqrt{53} \operatorname{cis} 74.05°$.

8 $6 \operatorname{cis}(120° - 30°) = 6 \operatorname{cis} 90° = 6i$.

9 $\frac{1}{3} \operatorname{cis}(305° - 65°) = \frac{1}{3} \operatorname{cis} 240° = \frac{1}{3}\big(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\big) = -\frac{1}{6} - \frac{\sqrt{3}}{6}i$.

10 $-1 + i$ implies $x = -1$ and $y = 1$, so (x, y) is in QII. We get $\theta = 135°$ and $r = \sqrt{2}$. Now,

$$(-1+i)^7 = \big(\sqrt{2} \operatorname{cis} 135°\big)^7 = 2^{7/2} \operatorname{cis}[7(135°)] = 2^{7/2} \operatorname{cis} 945° = 2^{7/2}\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right) = -8 - 8i.$$

11 $z^3 = -i$, so z is any cube root of $-i = \cos 270° + i \sin 270°$. The cube roots have the form $\cos\alpha + i\sin\alpha$ for

$$\alpha = \frac{270° + 360°k}{3} = 90° + 120°k$$

for $k = 0, 1, 2$. So $\alpha = 90°, 210°, 330°$, giving solutions $\operatorname{cis} 90°$, $\operatorname{cis} 210°$, $\operatorname{cis} 330°$.