# Math 122 Exam #1 Key (Spring 2024)

**1** So $(6x-4)+(8x-12)=180$, giving $14x=196$, and thus $x=14$. Measures of the angles are $80°$ and $100°$.

**2** $34°\,51'\,35'' = 34° + \left(\frac{51}{60}\right)^{\circ} + \left(\frac{35}{3600}\right)^{\circ} = \left(34 + \frac{51}{60} + \frac{35}{3600}\right)^{\circ} = 34.8597° \approx 34.860°.$

**3** Answer is $-84°\,42'\,50''$, obtained from

$$\begin{aligned}84.7138° &= 84° + (0.7138°)\left(\frac{60'}{1°}\right) = 84° + 42.828' = 84°\,42' + (0.828')\left(\frac{60''}{1'}\right)\\ &= 84°\,42' + 49.68'' = 84°\,42'\,49.68'' \approx 84°\,42'\,50''.\end{aligned}$$

**4** The third angle has measure

$$180° - (19°\,34'\,23'' + 41°\,5'\,11'') = 180° - 60°\,39'\,34'' = 179°\,59'\,60'' - 60°\,39'\,34'' = 119°\,20'\,26''.$$

**5** Here $r = \sqrt{x^2+y^2} = \sqrt{(-24)^2+(-7)^2} = 25$, so:

$$\sin\theta = -\frac{7}{25},\quad \cos\theta = -\frac{24}{25},\quad \tan\theta = \frac{7}{24},\quad \cot\theta = \frac{24}{7},\quad \sec\theta = -\frac{25}{24},\quad \csc\theta = -\frac{25}{7}.$$

**6** $\csc\theta = -2$ implies $\sin\theta = -\frac{1}{2}$, and $\cot\theta = \cos\theta/\sin\theta$. It remains to find $\cos\theta$. Since cosine is negative in quadrant III, from $\sin^2\theta + \cos^2\theta = 1$ comes $\cos\theta = -\sqrt{1-\sin^2\theta} = -\sqrt{\frac{3}{4}} = -\frac{\sqrt{3}}{2}$. Finally,

$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{-\sqrt{3}/2}{-1/2} = \sqrt{3}.$$

**7** $\cos\theta > 0$ in quadrants I and IV, while $\tan\theta < 0$ in quadrants II and IV. So we must be in quadrant IV.

Now, $x/r = \cos\theta = \sqrt{5}/8$, so we can let $x = \sqrt{5}$ and $r = 8$. This puts us on a circle of radius 8, which has equation $x^2+y^2=64$, and hence $y = -\sqrt{64-x^2} = -\sqrt{64-5} = -\sqrt{59}$ (note that $y<0$ in quadrant IV). Now,

$$\sin\theta = -\frac{\sqrt{59}}{8},\quad \cos\theta = \frac{\sqrt{5}}{8},\quad \tan\theta = -\frac{\sqrt{59}}{\sqrt{5}},\quad \cot\theta = -\frac{\sqrt{5}}{\sqrt{59}},\quad \sec\theta = \frac{8}{\sqrt{5}},\quad \csc\theta = -\frac{8}{\sqrt{59}}.$$

**8** We have $(5\theta + 2°) + (2\theta + 4°) = 90°$, or $\theta = 12°$.

**9** Using the 30-60 and 45-45 triangles, along with the Pythagorean theorem, we have

$$\cos 60° = \frac{a}{24} \quad\Rightarrow\quad a = 24\cos 60° = 24\cdot\frac{1}{2} = 12,$$

$$\sin 60^\circ = \frac{b}{24} \quad \Rightarrow \quad b = 24 \sin 60^\circ = 24 \cdot \frac{\sqrt{3}}{2} = 12\sqrt{3},$$

$c = \sqrt{b^2 + d^2} = 12\sqrt{6}$, and $d = b = 12\sqrt{3}$.

**10** 240° and 300°

**11** With a reciprocal identity, $\cos\theta = 0.861604813$, and so $\theta = 30.5027^\circ$.

**12** $A = 180^\circ - B - C = 38.3^\circ$,

$$\tan A = \frac{a}{b} \quad \Rightarrow \quad b = \frac{a}{\tan A} = \frac{28.1}{\tan 38.3^\circ} = 35.6 \text{ m},$$

$$\cos B = \frac{a}{c} \quad \Rightarrow \quad c = \frac{a}{\cos B} = \frac{28.1}{\cos 51.7^\circ} = 45.3 \text{ m}.$$

**13** Let $d$ be the distance. We have $\tan 32.5^\circ = \dfrac{252}{d}$, or $d = 396$ m.

**14** Drawing a line segment between the ships and from each ship to the port forms a right triangle. The first ship travels 42.5 km, the second 55 km. By the Pythagorean theorem the distance between the ships is $d = \sqrt{42.5^2 + 55^2} = 70$ km.