Whiteboard application is Xournal++

1.1 - Angles

An angle consists of two rays with a common endpoint, or two line segments with a common endpoint. Each ray/segment is a side of the angle. There is an initial side and a terminal side. We can think of an angle as being "swept out" by a single ray that rotates about its endpoint and then stops, with the initial direction of the ray being the initial side of the angle, and the ending directing being the terminal side.

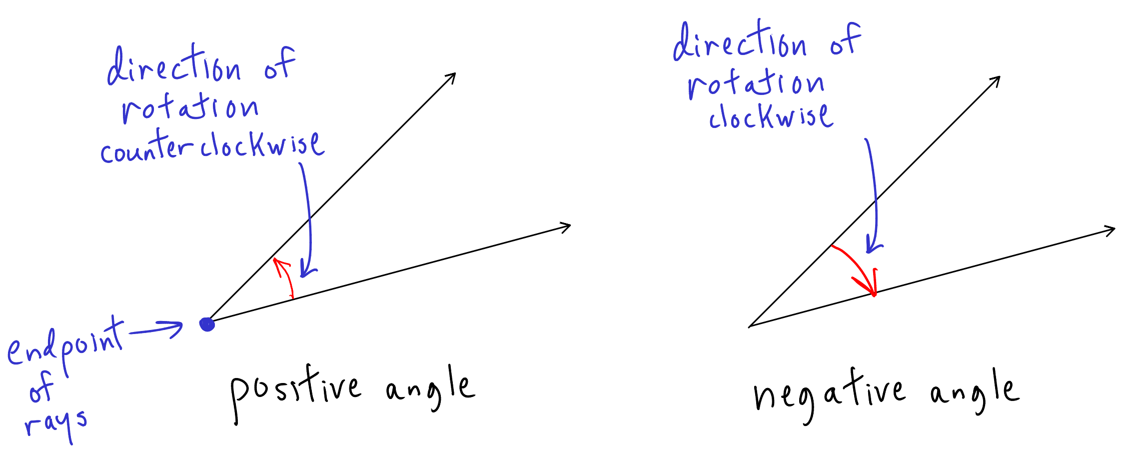
The extent of the rotation determines the measure of the angle that is "swept out" ("subtended" is the technical term). A counterclockwise rotation generates a positive measure, while a clockwise rotation generates a negative measure.

A full-circle rotation counterclockwise subtends a 360 degree (written 360°) angle, while a full-circle clockwise rotation subtends a -360° angle. A full-circle rotation is also called a complete rotation or a revolution.

So: 1° = 1 degree = 1/360 revolution

Also: 1' = 1 minute = 1/60 degree = $\left(1/60\right)^\circ$ (text: $\frac{1^\circ}{60}$)

1" = 1 second = 1/60 minute = $\left(1/60\right)'$ (text: $\frac{1'}{60}$)



Ex An angle can be said to have measure $58°27'$, which is shorthand for $58° + 27'$. We call $58°27'$ degree-minute format. ◻

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42 Add the angles, writing answer in degree-minute format: $110°25' + 32°55'$. Note that $60' = 1°$.

$$\begin{array}{rr} & 110°\ 25' \\ + & 32°\ 55' \\ \hline & 142°\ 80' \end{array} \rightarrow 142° + 80' \rightarrow 142° + 60' + 20' = 142° + 1° + 20'$$

$> 59'$, so we need to carry $60'$ into the degree column as $1°$.

$\rightarrow \boxed{143°20'}$ ◻

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44 Subtract the angles, writing answer in degree-minute format: $47°23' - 73°48'$

Since $73°48' > 47°23'$, we'll do the subtraction $73°48' - 47°23'$, and then append a negative sign to the answer...

$$\begin{array}{rr} & 73°\ 48' \\ - & 47°\ 23' \\ \hline & 26°\ 25' \end{array} \longrightarrow \text{Answer: } \boxed{-26°25'}$$ ◻

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50 Subtract $90° - 36°18'47''$, writing the answer in degree-minute-second format ($D°M'S''$).

$$\begin{array}{r} 90°\ 0'\ 0'' \\ 36°\ 18'\ 47'' \end{array} \longrightarrow \begin{array}{r} 89°\ 60'\ 0'' \\ 36°\ 18'\ 47'' \end{array} \longrightarrow \begin{array}{rr} & 89°\ 59'\ 60'' \\ - & 36°\ 18'\ 47'' \\ \hline & \boxed{53°\ 41'\ 13''} \end{array}$$ ◻

$$1'' = (1\cancel{''})\left(\frac{1/60\cancel{'}}{1\cancel{''}}\right)\left(\frac{1/60^\circ}{1\cancel{'}}\right) = \left(\frac{1}{60^2}\right)^\circ = \left(\frac{1}{3600}\right)^\circ \to 1'' = \left(\frac{1}{3600}\right)^\circ$$

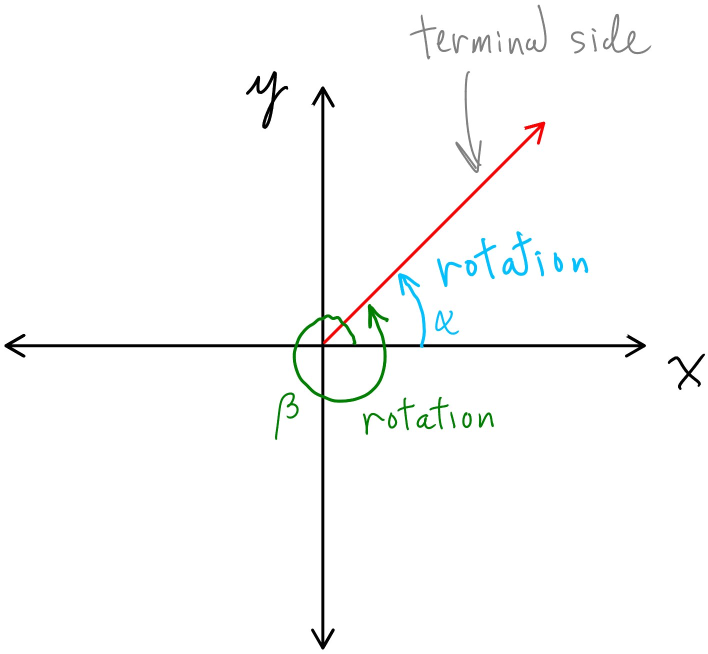
$\boxed{60}$ Convert the angle measure to decimal degrees: 38° 42' 18". Round to the nearest thousandth of a degree if applicable.

$$38^\circ 42' 18'' = 38^\circ + \left(\frac{42}{60}\right)^\circ + \left(\frac{18}{3600}\right)^\circ$$
$$= 38^\circ + 0.7^\circ + 0.005^\circ$$
$$= \boxed{38.705^\circ}$$

$\boxed{74}$ Convert to D°M′S″ format, rounding to the nearest second:
102.3771°

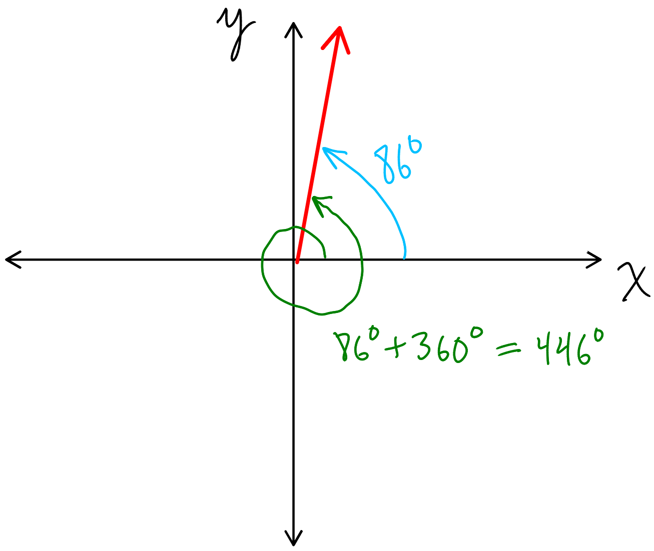
$$102^\circ + 0.3771^\circ = 102^\circ + (0.3771^\circ)\left(\frac{60'}{1^\circ}\right)$$
$$= 102^\circ + 22.626'$$
$$= 102^\circ + 22' + 0.626'$$
$$= 102^\circ 22' + (0.626')\left(\frac{60''}{1'}\right)$$
$$= 102^\circ 22' + 37.56''$$
$$= 102^\circ 22' 38''$$

Two angles with different rotations but the same terminal sides are called coterminal angles.



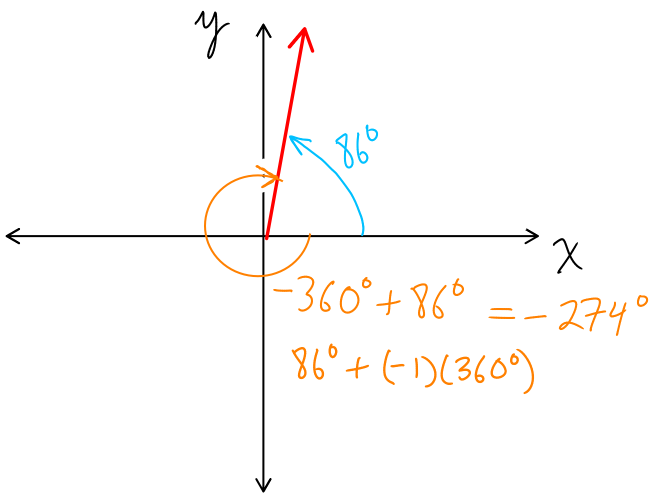
Angles α & β are coterminal.

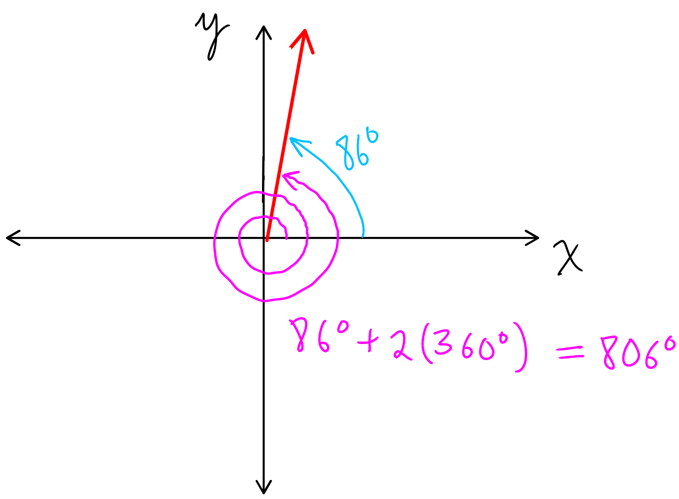
$\boxed{\text{Ex}}$ Find the angle of least positive measure that is coterminal with 86 °, but is not the same angle.



Answer: $\boxed{446^\circ}$

$\boxed{\text{Ex}}$ Find two more distinct angles that are coterminal with 86°.





Ex Write an expression that generates ALL angles that are coterminal with 86°.

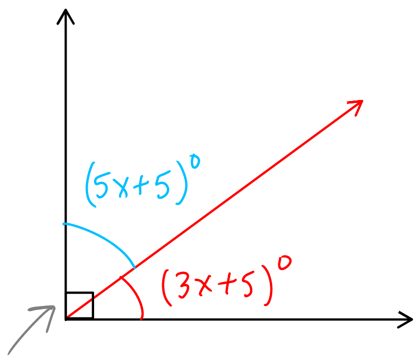
$\boxed{86^\circ + n(360^\circ), \text{ where } n \text{ is any integer.}}$

A quadrantal angle is an angle with measure $n(90^\circ)$, where n is any integer. So: 90°, 180°, 270°, 360°, -90°, -180°, etc.

Two angles are complementary if their measures add to 90°.

Two angles are supplementary if their measures add to 180°.

26 Find the measure of each angle.



□ indicates a right angle, measure 90°

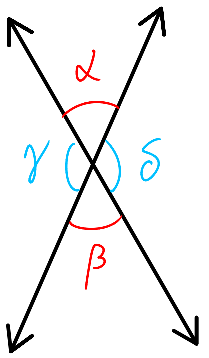
The angles $(3x+5)^\circ$ & $(5x+5)^\circ$ are complementary. Their measures are $3x+5$ & $5x+5$, and so:

$$(3x+5)+(5x+5)=90$$
$$8x+10=90$$
$$8x=80$$
$$x=10$$

So: $(3x+5)^\circ = (3\cdot 10+5)^\circ = \boxed{35^\circ}$

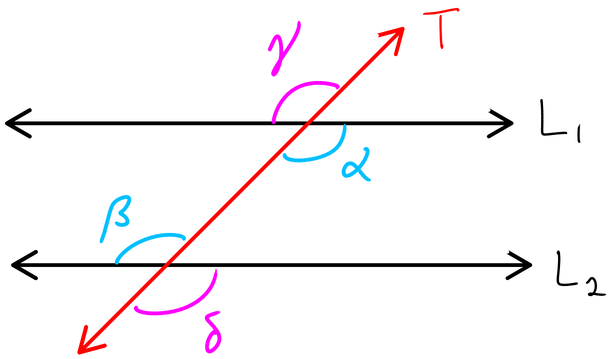
$(5x+5)^\circ = (5\cdot 10+5) = \boxed{55^\circ}$

1.2 - Angle Relationships and Similar Triangles



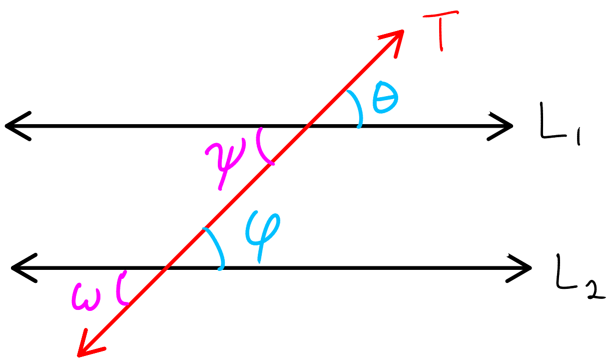
Angles α & β are vertical angles, as are γ & δ.

Fact: vertical angles have equal measures.



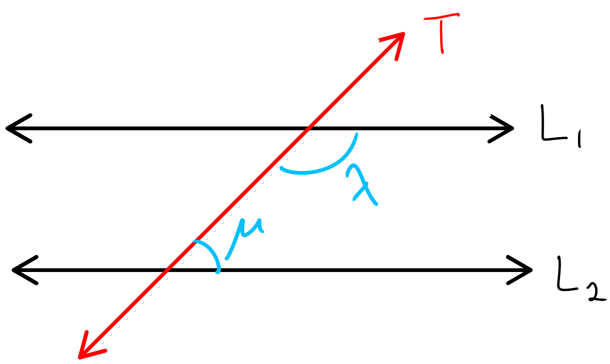
- Lines L_1 & L_2 are parallel.
- Line T is a transversal line.
- Angles α & β are called alternate interior angles.

Fact: alternate interior angles have equal measure.



- Angles γ & δ are called alternate exterior angles.

Fact: alternate exterior angles have equal measure.



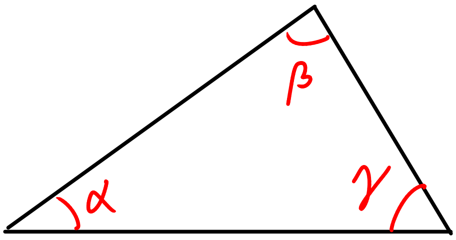
- Angles θ & φ are called corresponding angles, as are ψ & ω.

Fact: corresponding angles have equal measure.

So the measures of λ & μ sum to $180°$

Fact: Interior angles on the same side of a transversal are supplementary.

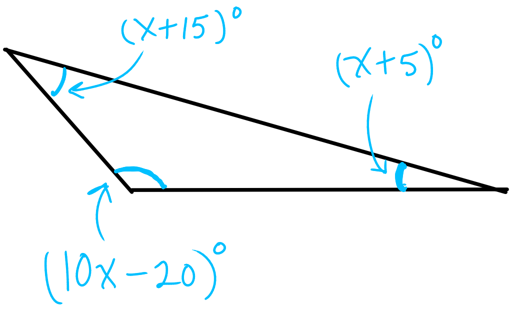
Notation: The measure of an angle θ is sometimes written as $m\angle\theta$. So: $m\angle\lambda + m\angle\mu = 180°$.



Angles α, β, γ are the interior angles of the triangle.

Fact: $m\angle\alpha + m\angle\beta + m\angle\gamma = 180°$

$\boxed{14}$ Find the measure of the interior angles.



We have $(x+15)+(x+5)+(10x-20)=180$

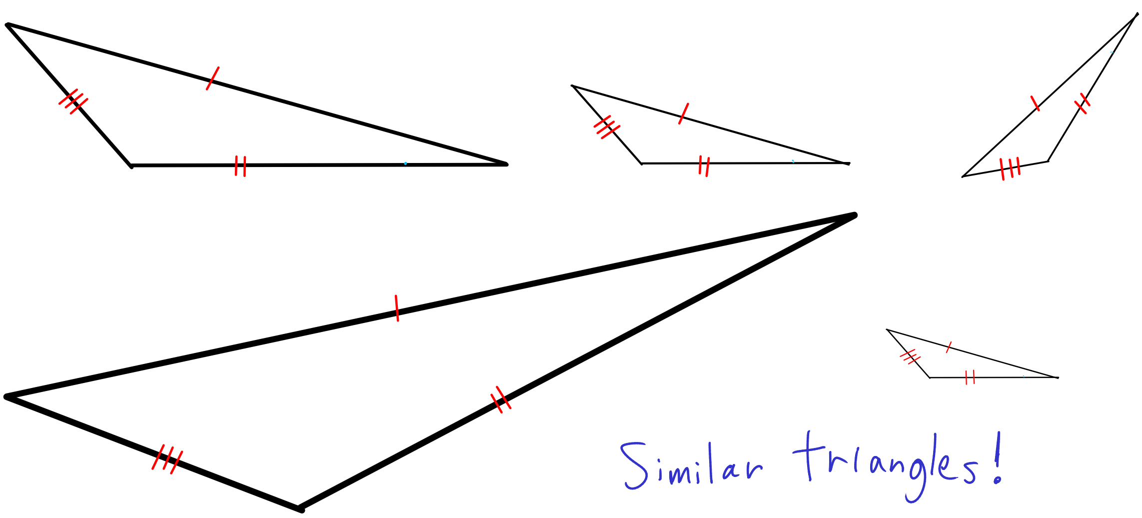
$$12x = 180$$

$$x = 15.$$

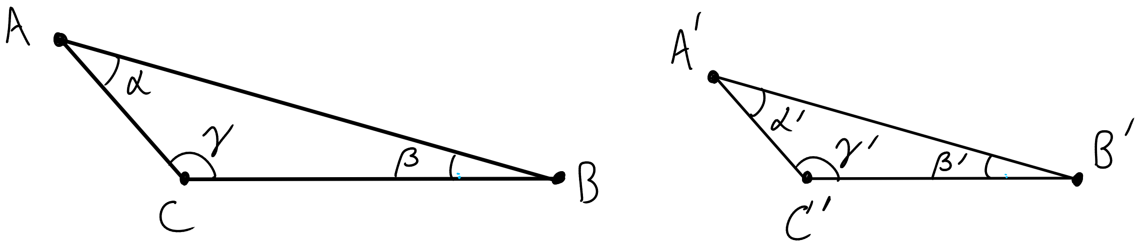
Measures are $(x+15)° = (15+15)° = \boxed{30°}$

$(x+5)° = (15+5)° = \boxed{20°}$

$(10x-20)° = (10\cdot 15-20)° = \boxed{130°}$ ▫



Similar triangles!



We have two similar triangles: $\triangle ABC$ and $\triangle A'B'C'$

Corresponding sides: AB & A'B', AC & A'C', BC & B'C'.

Corresponding angles: α & α', β & β', γ & γ'.

We'll let $\overline{AB}$ or AB denote the line segment from point A to point B.
The length of AB we'll indicate by $\ell(AB)$

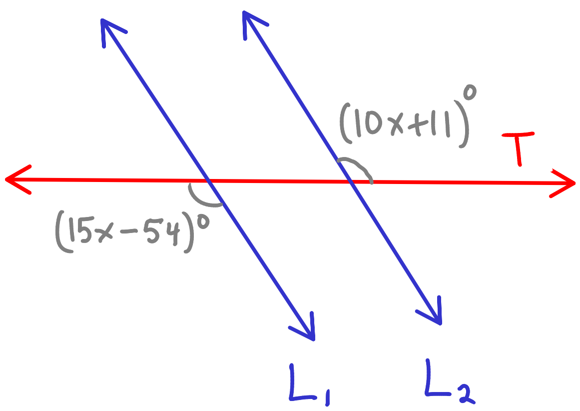
The triangles are similar if and only if the following holds:

1) $m\angle\alpha = m\angle\alpha', \; m\angle\beta = m\angle\beta', \; m\angle\gamma = m\angle\gamma'.$

2) $\dfrac{\ell(AB)}{\ell(A'B')} = \dfrac{\ell(AC)}{\ell(A'C')} = \dfrac{\ell(BC)}{\ell(B'C')}.$

In words:
1) The measures of corresponding angles are equal.
2) The ratios of corresponding sides are equal.

$\boxed{22}$ Find the measure of each marked angle.



The angles are alternate exterior angles, which are equal, and so we have the equation

$10x+11 = 15x-54$

$5x = 65$

$x = 13$

Thus we have $(15x-54)^\circ = 141^\circ$. The angles are each $\boxed{141^\circ}$ in measure.

$\boxed{26}$ $136°50'$ & $41°38'$ are the measures of two interior angles of a triangle. Find the measure of the 3rd angle.

Let θ be the 3rd angle. Then:

$$136°50' + 41°38' + m\angle\theta = 180°$$

$$m\angle\theta = 180° - \underbrace{(136°50' + 41°38')}$$

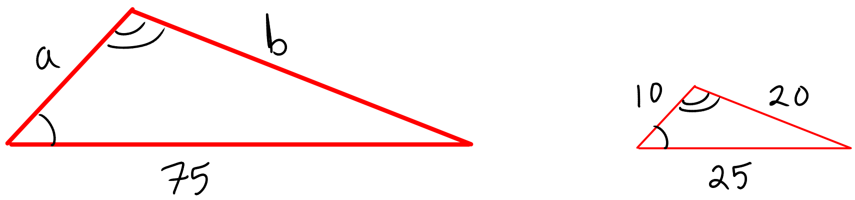
$$\begin{array}{r} 136°50' \\ +\ \ 41°38' \\ \hline 177°88' \end{array} \longrightarrow \begin{array}{l} 177° + 60' + 28' \\ 177° + 1° + 28' \\ 178°28' \end{array}$$

$$m\angle\theta = 180° - 178°28'$$

$$= 179°60' - 178°28'$$

$$= \boxed{1°32'}$$

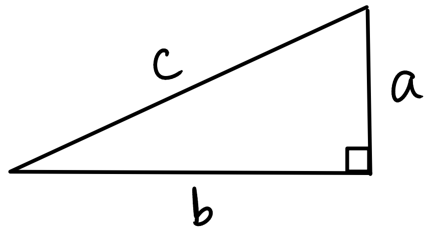
$\boxed{54}$ Find the unknown side lengths in the pair of similar triangles:



$$\frac{a}{10} = \frac{b}{20} = \frac{75}{25} \Rightarrow \frac{a}{10} = 3 \;\&\; \frac{b}{20} = 3 \Rightarrow$$

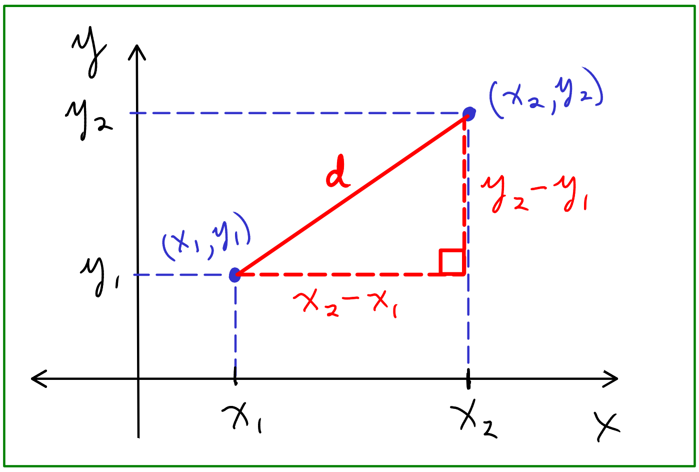
$$\boxed{a = 30 \;\&\; b = 60}$$

1.3 - Trigonometric Functions



Pythagorean Theorem: $a^2 + b^2 = c^2$

Using the Pythagorean theorem we can derive the distance formula, which finds the distance between two points (x_1, y_1) & (x_2, y_2) on the xy-plane.

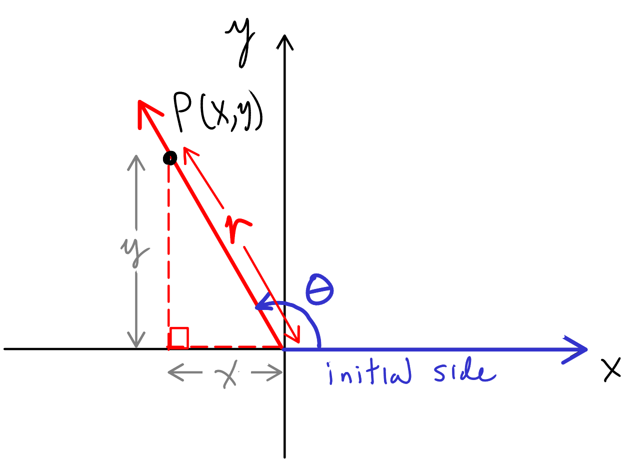


$$(x_2 - x_1)^2 + (y_2 - y_1)^2 = d^2$$

$$\boxed{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

(Distance Formula)

Let θ be an angle in standard position; that is, the initial side of the angle is on the positive x-axis. Let P be a point on the terminal side of θ, with coordinates (x,y). Note: P should not be at the origin, where the initial and terminal sides meet.



Note that here we have $x < 0$ & $y > 0$. So x is not the length of the horizontal side of the triangle. We call it a "directed length"!

Pythagorean theorem: $r^2 = x^2 + y^2 \implies \boxed{r = \sqrt{x^2 + y^2}}$ (Note: $r > 0$)

We define the six trigonometric functions:

sine function (symbol sin): $\sin\theta = \frac{y}{r}$

cosine function (symbol cos): $\cos\theta = \frac{x}{r}$

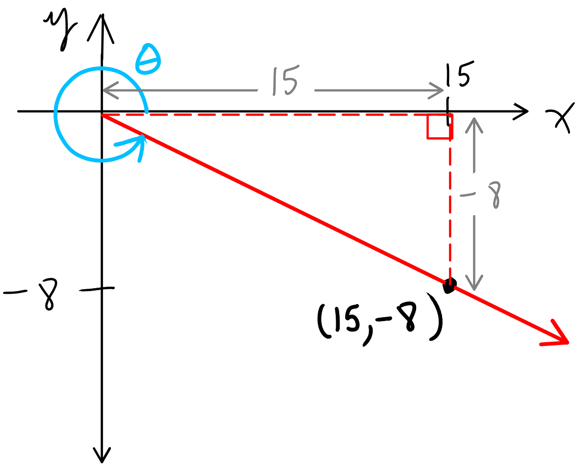
tangent function (symbol tan): $\tan\theta = \frac{y}{x}$

cosecant function (symbol csc): $\csc\theta = \frac{r}{y}$

secant function (symbol sec): $\sec\theta = \frac{r}{x}$

cotangent function (symbol cot): $\cot\theta = \frac{x}{y}$

$\boxed{16}$ Sketch an angle θ having least positive measure and the point $(15,-8)$ on its terminal side. Find the values of the six trigonometric functions for θ.



Here we have:

$x = 15$

$y = -8$

$r = \sqrt{15^2 + (-8)^2} = \sqrt{289} = 17$

$\sin\theta = \frac{y}{r} = \boxed{\frac{-8}{17}}$, $\cos\theta = \frac{x}{r} = \boxed{\frac{15}{17}}$,

$\tan\theta = \frac{y}{x} = \boxed{\frac{-8}{15}}$, $\cot\theta = \frac{x}{y} = \boxed{\frac{15}{-8}}$,

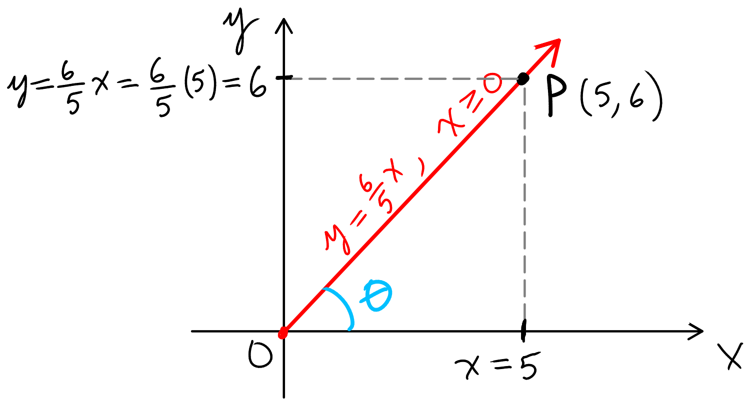
$\csc\theta = \frac{r}{y} = \boxed{\frac{17}{-8}}$, $\sec\theta = \frac{r}{x} = \boxed{\frac{17}{15}}$.

56 The terminal side of an angle is given by:

$$6x-5y=0 \text{ for } x\geq 0.$$

Sketch the least positive such angle, denoted by θ. Find the values of the six trigonometric functions of θ.

$$6x-5y=0 \Rightarrow 5y=6x \Rightarrow y=\frac{6}{5}x \text{ for } x\geq 0.$$



Here we have:

$x=5$ (our choice)

$y=6$

$r=\sqrt{5^2+6^2}=\sqrt{61}$

$\sin\theta=\frac{y}{r}=\frac{6}{\sqrt{61}}=\boxed{\frac{6\sqrt{61}}{61}}$, $\tan\theta=\frac{y}{x}=\boxed{\frac{6}{5}}$

$\cos\theta=\frac{x}{r}=\frac{5}{\sqrt{61}}=\boxed{\frac{5\sqrt{61}}{61}}$, $\csc\theta=\frac{r}{y}=\boxed{\frac{\sqrt{61}}{6}}$

$\sec\theta=\frac{r}{x}=\boxed{\frac{\sqrt{61}}{5}}$, $\cot\theta=\frac{x}{y}=\boxed{\frac{5}{6}}$ ■

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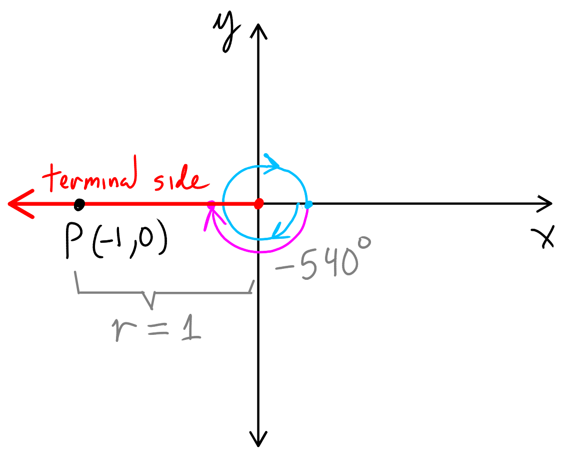
74 Find $\sec(-540^\circ)$.

Note: $-540^\circ=(-360^\circ)+(-180^\circ)$, so -540° is coterminal with -180°



Here $x=-1$, $y=0$, and

$r=\sqrt{x^2+y^2}=\sqrt{(-1)^2+0^2}=\sqrt{1}=1.$

$\sec(-540^\circ)=\frac{x}{r}=\frac{-1}{1}=\boxed{-1}$ ■

1.4 - Using the Definitions of the Trigonometric Functions

sine function (symbol sin): $\sin\theta = \frac{y}{r}$

cosine function (symbol cos): $\cos\theta = \frac{x}{r}$

tangent function (symbol tan): $\tan\theta = \frac{y}{x}$

cosecant function (symbol csc): $\csc\theta = \frac{r}{y}$

secant function (symbol sec): $\sec\theta = \frac{r}{x}$

cotangent function (symbol cot): $\cot\theta = \frac{x}{y}$

An identity is an equation which is satisfied for all values of the variable for which both sides of the equation are defined.

Ex For instance, $\frac{(x+1)(x-1)}{x+1} = x-1$ is an identity:

Both sides are defined for all x ≠ -1, and for all such values of x the equation is satisfied; that is, the left side's value matches the right side's value. Solution set consists of all reals except for -1, or in interval notation: $(-\infty,-1)\cup(-1,\infty)$.

We find some trigonometric identities in this section, using the definitions of the trigonometric functions above.

$\csc\theta = \frac{r}{y} = \frac{1}{\frac{y}{r}} = \frac{1}{\sin\theta}$, and so $\sin\theta = \frac{1}{\csc\theta}$.

$\sec\theta = \frac{r}{x} = \frac{1}{\frac{x}{r}} = \frac{1}{\cos\theta}$, and so $\cos\theta = \frac{1}{\sec\theta}$.

$\cot\theta = \frac{x}{y} = \frac{1}{\frac{y}{x}} = \frac{1}{\tan\theta}$, and so $\tan\theta = \frac{1}{\cot\theta}$.

We have the reciprocal identities:

$$\csc\theta = \frac{1}{\sin\theta}, \quad \sec\theta = \frac{1}{\cos\theta}, \quad \cot\theta = \frac{1}{\tan\theta}$$

$$\sin\theta = \frac{1}{\csc\theta}, \quad \cos\theta = \frac{1}{\sec\theta}, \quad \tan\theta = \frac{1}{\cot\theta}$$

- $\dfrac{\sin\theta}{\cos\theta} = \dfrac{y/r}{x/r} = \dfrac{y}{x} = \tan\theta$ & $\dfrac{\cos\theta}{\sin\theta} = \dfrac{1}{\tan\theta} = \cot\theta$

Quotient Identities

$$\frac{\sin\theta}{\cos\theta} = \tan\theta, \quad \frac{\cos\theta}{\sin\theta} = \cot\theta$$

- Recall that $r = \sqrt{x^2+y^2}$, so $r^2 = x^2+y^2 \Rightarrow \dfrac{x^2+y^2}{r^2} = 1 \Rightarrow$ $\dfrac{x^2}{r^2} + \dfrac{y^2}{r^2} = 1 \Rightarrow \left(\dfrac{x}{r}\right)^2 + \left(\dfrac{y}{r}\right)^2 = 1 \Rightarrow \cos^2\theta + \sin^2\theta = 1$.

Note: $\sin^2\theta$ means $(\sin\theta)^2$, while $\sin\theta^2$ means $\sin(\theta^2)$

- Now, $\cos^2\theta + \sin^2\theta = 1 \Rightarrow \dfrac{\cos^2\theta + \sin^2\theta}{\cos^2\theta} = \dfrac{1}{\cos^2\theta} \Rightarrow$ $1 + \left(\dfrac{\sin\theta}{\cos\theta}\right)^2 = \left(\dfrac{1}{\cos\theta}\right)^2 \Rightarrow 1 + \tan^2\theta = \sec^2\theta$.

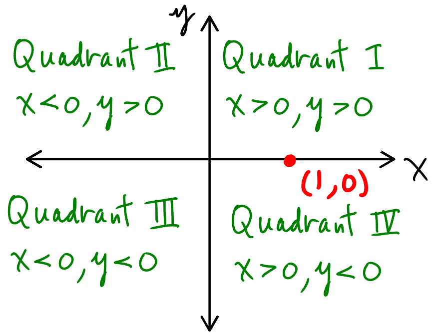
- Now, $\cos^2\theta + \sin^2\theta = 1 \Rightarrow \dfrac{\cos^2\theta + \sin^2\theta}{\sin^2\theta} = \dfrac{1}{\sin^2\theta} \Rightarrow$ $\left(\dfrac{\cos\theta}{\sin\theta}\right)^2 + 1 = \left(\dfrac{1}{\sin\theta}\right)^2 \Rightarrow \cot^2\theta + 1 = \csc^2\theta$.

Pythagorean Identities:

$$\cos^2\theta + \sin^2\theta = 1, \quad 1 + \tan^2\theta = \sec^2\theta, \quad \cot^2\theta + 1 = \csc^2\theta$$

$\boxed{20}$ Find $\sin\theta$, given that $\csc\theta = \frac{\sqrt{24}}{3}$.

$$\sin\theta = \frac{1}{\csc\theta} = \frac{1}{\frac{\sqrt{24}}{3}} = \frac{3}{\sqrt{24}} = \frac{3\sqrt{24}}{24} = \boxed{\frac{\sqrt{24}}{8}}$$



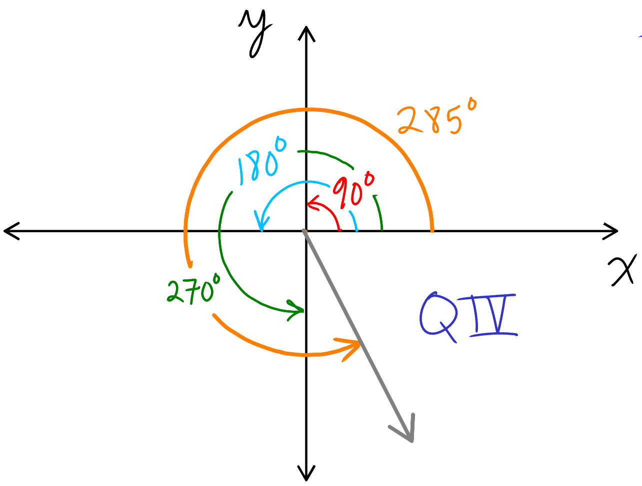
If x=0 or y=0, then we're not in any of the four quadrants.

So the point (1,0) is in <u>no</u> quadrant.

$\boxed{36}$ Find the signs (positive or negative) of the trigonometric functions of the angle 1005°.

We need to determine the quadrant that the terminal side of the angle 1005° is in, if any.

$1005° = 2(360°) + 285°$, so $1005°$ is coterminal with $285°$



285° is in quadrant IV (can denote by QIV)

So $x > 0$ & $y < 0$.

Since $r > 0$ is always the case, we have:

$\sin\theta = \frac{y}{r} < 0$ since $y < 0$ $\longrightarrow$ $\sin\theta$ is negative

$\cos\theta = \frac{x}{r} > 0$ since $x > 0$ $\longrightarrow$ $\cos\theta$ is positive

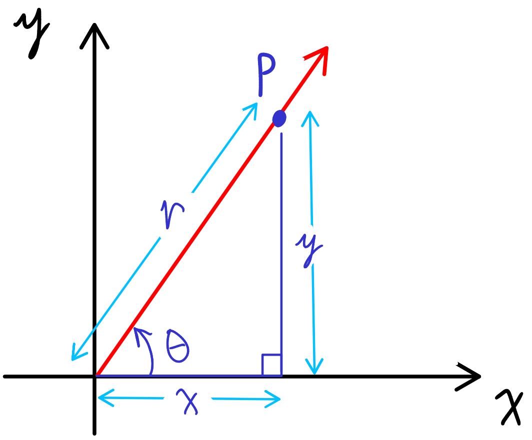
$\tan\theta = \frac{y}{x} < 0$ $\longrightarrow$ $\tan\theta$ is negative

$\csc\theta = \frac{r}{y} < 0 \longrightarrow \csc\theta$ is negative

$\sec\theta = \frac{r}{x} > 0 \longrightarrow \sec\theta$ is positive

$\cot\theta = \frac{x}{y} < 0 \longrightarrow \cot\theta$ is negative.

$\boxed{48}$ In what quadrant(s) can the terminal side of θ be in to have $\cot\theta < 0$ & $\sec\theta < 0$?



- Since $\cot\theta = \frac{x}{y}$, we need $\frac{x}{y} < 0$. Possibilities are:

  $x < 0, y > 0$ (QII)

  $x > 0, y < 0$ (QIV)

- Since $\sec\theta = \frac{r}{x} < 0$, we need $x < 0$ (r is always positive), which is the case in QII & QIII.

The only quadrant where both given inequalities are satisfied is:

$\boxed{\text{QII}}$

$\boxed{54}$ Is $\sin\theta = 3$ possible or not?

$\sin\theta = \frac{y}{r} = \frac{y}{\sqrt{x^2+y^2}}$.

Note: $\sqrt{x^2+y^2} \geq \sqrt{y^2} = |y|$, and so $\sqrt{x^2+y^2} \geq y$, and then $\frac{y}{\sqrt{x^2+y^2}} \leq 1$. Hence $\sin\theta \leq 1$, and $\sin\theta = 3$ is impossible!

Note: For problems like #54, you can just use the table that gives the ranges of the trigonometric functions that is on page 151 of the textbook. Then #54 is a 5-second problem.

$\boxed{68}$ Find $\sec\theta$, given: $\tan\theta = \frac{\sqrt{7}}{3}$ & θ is in Q III.

- $\frac{\sqrt{7}}{3} = \tan\theta = \frac{y}{x}$, so $\frac{y}{x} = \frac{\sqrt{7}}{3}$.
- We can take $|x| = 3$ & $|y| = \sqrt{7}$.
- Since θ is in Q III, we have $x < 0$ & $y < 0$.
- So we can let $x = -3$ & $y = -\sqrt{7}$.

Now, $\sec\theta = \frac{r}{x} = \frac{\sqrt{x^2+y^2}}{x} = \frac{\sqrt{(-3)^2 + (-\sqrt{7})^2}}{-3} = \frac{\sqrt{9+7}}{-3} \Rightarrow$

$\sec\theta = \frac{\sqrt{16}}{-3} = \boxed{-\frac{4}{3}}$

$\boxed{74}$ Given: $\cos\theta = -\frac{3}{5}$ & θ is in Q III.

Find the trigonometric functions of θ.

- We have $\frac{x}{r} = \cos\theta = -\frac{3}{5}$, and so we can let $x = -3$ & $r = 5$.
- Now $r = \sqrt{x^2+y^2} \Rightarrow x^2 + y^2 = r^2 \Rightarrow y^2 = r^2 - x^2 \Rightarrow$
  $\sqrt{y^2} = \sqrt{r^2 - x^2} \Rightarrow |y| = \sqrt{r^2-x^2} \Rightarrow y = \pm\sqrt{r^2-x^2}$
- Since θ is in Q III we must have $y < 0$, and so $y = -\sqrt{r^2-x^2}$
- Then: $y = -\sqrt{r^2-x^2} = -\sqrt{5^2 - (-3)^2} = -\sqrt{16} = -4 \Rightarrow y = -4$
- Finally: $\sin\theta = \frac{y}{r} = \boxed{-\frac{4}{5}}$

  $\tan\theta = \frac{y}{x} = \frac{-4}{-3} = \boxed{\frac{4}{3}}$

  $\sec\theta = \frac{r}{x} = \boxed{-\frac{5}{3}}$

  $\csc\theta = \frac{r}{y} = \boxed{-\frac{5}{4}}$

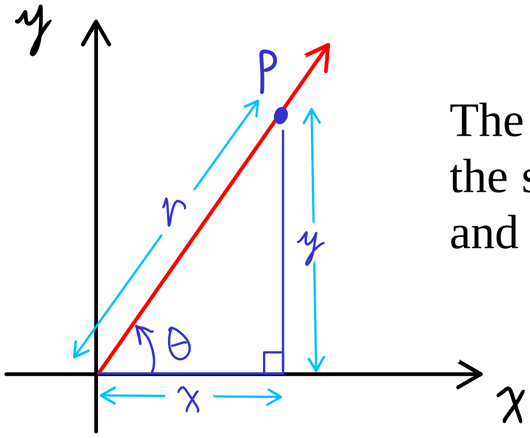
  $\cot\theta = \frac{x}{y} = \frac{-3}{-4} = \boxed{\frac{3}{4}}$

Could use reciprocal identities

2.1 - Trigonometric Functions of Acute Angles

Recall that an angle θ is acute if $0^\circ < \theta < 90^\circ$.

Such an angle θ in standard position is illustrated below.



The side of length y is called the side opposite θ, the side of length x is the side adjacent to θ, and the side of length r is the hypotenuse.

θ is acute!

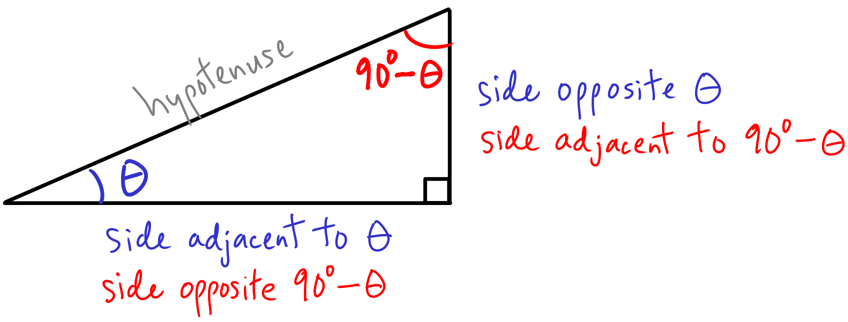
Based on the definitions of the trigonometric functions, we have:

$$\sin\theta = \frac{y}{r} = \frac{\text{side opposite } \theta}{\text{hypotenuse}}, \qquad \csc\theta = \frac{r}{y} = \frac{\text{hypotenuse}}{\text{side opposite } \theta}$$

$$\cos\theta = \frac{x}{r} = \frac{\text{side adjacent to } \theta}{\text{hypotenuse}}, \qquad \sec\theta = \frac{r}{x} = \frac{\text{hypotenuse}}{\text{side adjacent to } \theta}$$

$$\tan\theta = \frac{y}{x} = \frac{\text{side opposite } \theta}{\text{side adjacent to } \theta}, \qquad \cot\theta = \frac{x}{y} = \frac{\text{side adjacent to } \theta}{\text{side opposite } \theta}$$

The six trigonometric functions come in three cofunction pairs. We derive two of six cofunction identities...



$$\sin\theta = \frac{\text{side opposite } \theta}{\text{hypotenuse}} = \frac{\text{side adjacent to } 90^\circ-\theta}{\text{hypotenuse}} = \cos(90^\circ-\theta)$$

$$\tan\theta = \frac{\text{side opposite } \theta}{\text{side adjacent to } \theta} = \frac{\text{side adjacent to } 90^\circ-\theta}{\text{side opposite } 90^\circ-\theta} = \cot(90^\circ-\theta)$$

Cofunction Identities

For any acute angle θ:

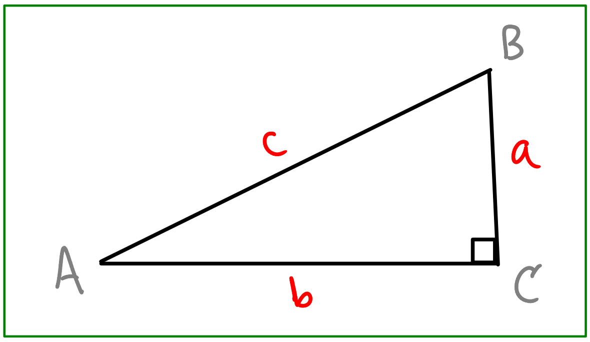
$\sin\theta = \cos(90^\circ-\theta)$, $\sec\theta = \csc(90^\circ-\theta)$

$\cos\theta = \sin(90^\circ-\theta)$, $\csc\theta = \sec(90^\circ-\theta)$

$\tan\theta = \cot(90^\circ-\theta)$, $\cot\theta = \tan(90^\circ-\theta)$

The cofunction identities show the relationship between the sine and COsine, secant and COsecant, and tangent and COtangent functions.

16



- Given: $b=8$, $c=11$.
- Find: (i) Side a
  (ii) The six trig. functions for angle B.

We find a using the Pythagorean theorem: $a^2+b^2=c^2 \Rightarrow$

$a=\sqrt{c^2-b^2}$ (note $|a|=a$ since a is the length of a side)

So $a=\sqrt{11^2-8^2} = \sqrt{121-64} = \sqrt{57} \Rightarrow \boxed{a=\sqrt{57}}$

$$\sin B = \frac{\text{side opposite } B}{\text{hypotenuse}} = \frac{b}{c} = \boxed{\frac{8}{11}}$$

$$\cos B = \frac{\text{side adjacent to } B}{\text{hypotenuse}} = \frac{a}{c} = \boxed{\frac{\sqrt{57}}{11}}$$

$$\tan B = \frac{\text{side opposite } B}{\text{side adjacent to } B} = \frac{b}{a} = \frac{8}{\sqrt{57}} = \boxed{\frac{8\sqrt{57}}{57}}$$

$$\csc B = \frac{1}{\sin B} = \boxed{\frac{11}{8}}, \quad \sec B = \frac{1}{\cos B} = \frac{11}{\sqrt{57}} = \boxed{\frac{11\sqrt{57}}{57}},$$

$$\cot B = \frac{1}{\tan B} = \boxed{\frac{\sqrt{57}}{8}}$$

$\boxed{26}$ Write $\tan 25.4^\circ$ in terms of its cofunction.

The cofunction of tan is cot, and we have the cofunction identity

$\tan\theta = \cot(90^\circ - \theta)$, so with $\theta = 25.4^\circ$ we get

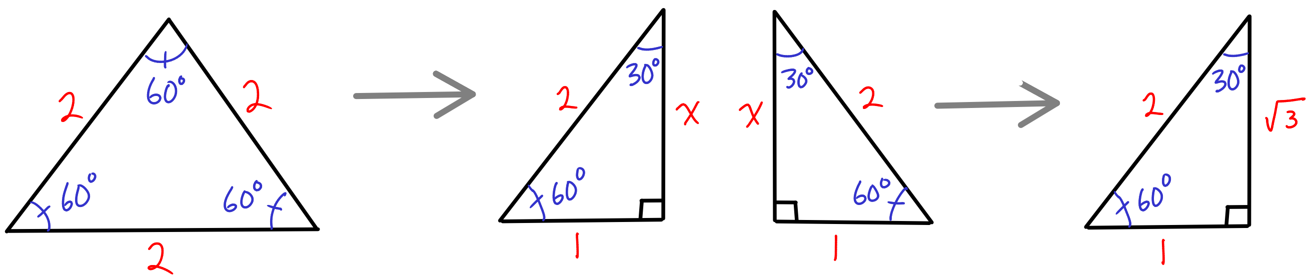
$\tan 25.4^\circ = \cot(90^\circ - 25.4^\circ) = \boxed{\cot(64.6^\circ)}$

- Recall that $\cos\theta = \sin(90^\circ - \theta)$, where $\theta + (90^\circ - \theta) = 90^\circ$.
- But more generally we have $\cos\alpha = \sin\beta$ whenever α & β are acute angles such that $\alpha + \beta = 90^\circ$.
- There are other ways to satisfy $\cos\alpha = \sin\beta$, but having $\alpha + \beta = 90^\circ$ is one way...

$\boxed{32}$ Find one solution θ to the equation, assuming all angles are acute: $\cos\theta = \sin(2\theta - 30^\circ)$

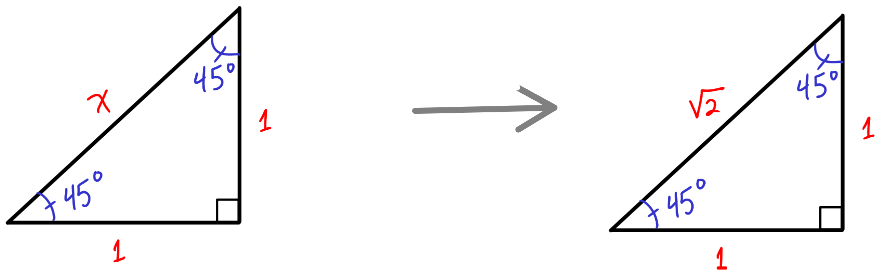
- We have $\cos\alpha = \sin(90^\circ - \alpha)$ for any acute angle α.
  Note: $\alpha + (90^\circ - \alpha) = 90^\circ$.
- So we have $\theta + (2\theta - 30^\circ) = 90^\circ$
- Solve for θ: $3\theta = 120^\circ \Rightarrow \boxed{\theta = 40^\circ}$

Aside from the quadrantal angles, there are a number of "special angles" for which the values of the trigonometric functions can be determined exactly. They derive from two special triangles, the first of which is the "30-60 triangle."



Equilateral triangle | $x^2 + 1^2 = 2^2 \Rightarrow x = \sqrt{3}$ | 30-60 triangle

The other special triangle is the "45-45 triangle."



$$\boxed{1^2+1^2 = x^2 \Rightarrow x = \sqrt{2}}$$

$\boxed{\text{45-45 triangle}}$

With these triangles we get the following:

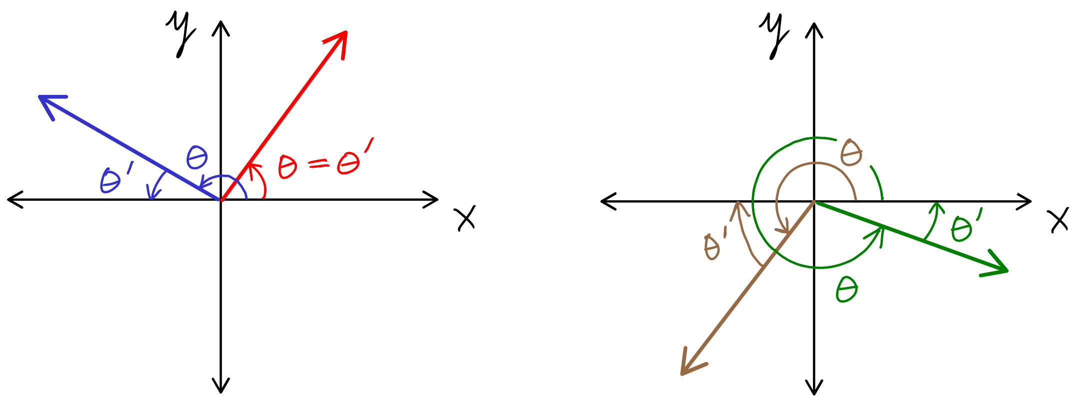
θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\csc\theta$
$30°$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2
$45°$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	1	$\sqrt{2}$	$\sqrt{2}$
$60°$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$

Irrational denominators are not rationalized so as not to obscure the relationship the values in the table have to the special triangles.

2.2 - Trigonometric Functions of Non-Acute Angles

Definition The reference angle for any nonquadrantal angle θ, denoted by θ', is the acute angle formed by the terminal side of θ and the x-axis.

The definition implies that $0° < \theta' < 90°$ for any nonquadrantal angle θ.

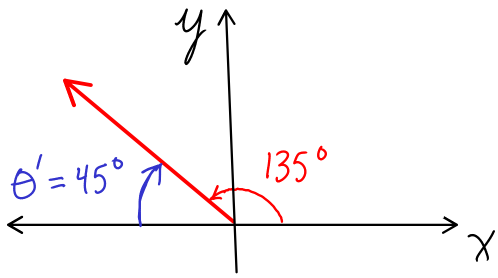


24 Find exact values of the six trig. functions of $\theta = 495°$.

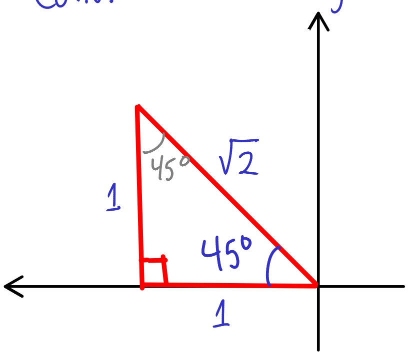
- Find an angle in the interval [0°,360°) that is coterminal with θ.

  $495° - 360° = 135°$.

- Find reference angle for $135°$, which is $\theta' = 180° - 135° = 45°$



- Construct a triangle having terminal side of θ' as the hypotenuse



- Evaluate the trig. functions of θ', with the correct sign (+ or −) depending on the quadrant the terminal side of θ' is in.

$$\sin 495^\circ = \sin 135^\circ = +\sin 45^\circ = \boxed{\frac{1}{\sqrt{2}}}$$

$$\cos 495^\circ = \cos 135^\circ = -\cos 45^\circ = \boxed{-\frac{1}{\sqrt{2}}}$$

$$\tan 495^\circ = \tan 135^\circ = -\tan 45^\circ = \boxed{-1}$$

$$\cot 495^\circ = \frac{1}{\tan 495^\circ} = \frac{1}{-1} = \boxed{-1}$$

$$\sec 495^\circ = \frac{1}{\cos 495^\circ} = \boxed{-\sqrt{2}}$$

$$\csc 495^\circ = \frac{1}{\sin 495^\circ} = \boxed{\sqrt{2}} \quad \blacksquare$$

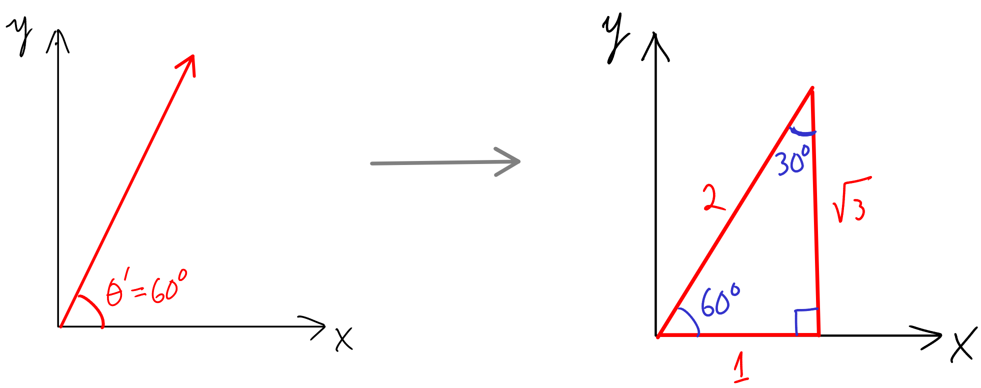
$\boxed{32}$ Find exact values of the six trig. functions of $\theta = -1020^\circ$

- Find an angle in the interval [0°,360°) that is coterminal with θ.

$$-1020^\circ + 3(360^\circ) = 60^\circ$$

- Find the reference angle for 60°, which is also 60°. That is, $\theta' = 60^\circ$.

- Construct a triangle having hypotenuse on the terminal side of the reference angle of $\theta' = 60^\circ$



- Evaluate the six trigonometric functions of $\theta' = 60^\circ$, appending the correct sign (+ or -) depending on the quadrant the terminal side of θ' is in.

$$\sin(-1020^\circ) = +\sin 60^\circ = \boxed{\frac{\sqrt{3}}{2}}$$

$\cos(-1020°) = +\cos 60° = \boxed{\frac{1}{2}}$

$\tan(-1020°) = +\tan 60° = \boxed{\sqrt{3}}$

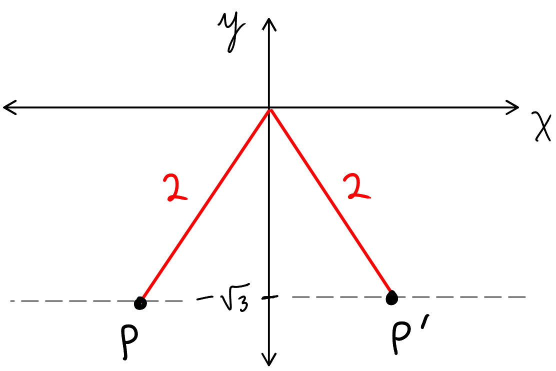
$\cot(-1020°) = \dfrac{1}{\tan(-1020°)} = \boxed{\frac{1}{\sqrt{3}}}$

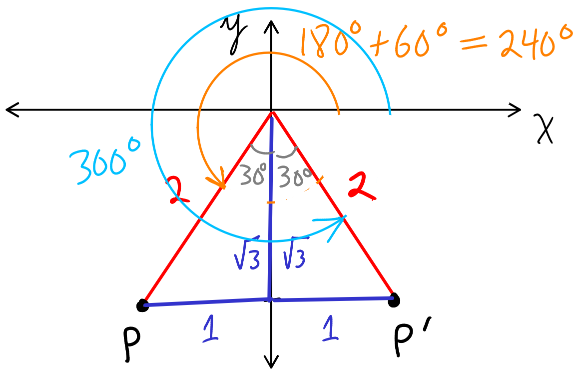
$\sec(-1020°) = \dfrac{1}{\cos(-1020°)} = \boxed{2}$

$\csc(-1020°) = \dfrac{1}{\sin(-1020°)} = \boxed{\frac{2}{\sqrt{3}}}$

$\boxed{68}$ $\sin\theta = -\dfrac{\sqrt{3}}{2}$. Find all values of θ in the interval $[0°, 360°)$ that satisfy the equation.

$\sin\theta = \dfrac{y}{r} = \dfrac{-\sqrt{3}}{2}$, so $y = -\sqrt{3}$ & $r = 2$.





Solutions: $\boxed{240° \ \& \ 300°}$

Exam #1 on February 29 (not February 27), on Chapters 1 and 2.

2.3 - Approximations of Trigonometric Functions

Make sure your calculator is in degree mode!

Ex Approximate $\cos(141° 17')$.

If a calculator allows entering an angle in D°M′S″ format, then we can just key it in and get a decimal approximation of $\cos(141° 17')$.

If a calculator doesn't allow it, convert the angle to degrees. We'll assume the given angle is exact in this section, and give approximations to trigonometric functions correct to six decimal places.

$$141° 17' = 141° + \left(\frac{17}{60}\right)^{\circ} = 141.28\overline{3}^{\circ}$$

$$\cos(141° 17') = \cos 141.28\overline{3}^{\circ} = -0.780248498 \approx \boxed{-0.780248}$$

Recall that the inverse of a function f is denoted by f^{-1}.

Warning: while $x^{-1} = \frac{1}{x}$, to write $f^{-1} = \frac{1}{f}$ is wrong!

Recall from algebra that f^{-1} ("f inverse" or "the inverse of f") relates to function f as follows:

$$\boxed{\begin{array}{l} f(x) = y \iff f^{-1}(y) = x \\ \text{for all } x \text{ in the domain of } f \\ \text{and } y \text{ in the range of } f. \end{array}}$$

A scientific calculator has three inverse trigonometric functions:

$$\sin^{-1}, \quad \cos^{-1}, \quad \tan^{-1}$$

The inverse sine function $\sin^{-1}$ relates to the sine function thusly:

$$\boxed{\sin\theta = t \iff \sin^{-1} t = \theta}$$

Similar relationships hold for the inverse cosine and inverse tangent functions (as well as the inverses of cosecant, secant, and cotangent).

$\boxed{36}$ Find a value of θ in the interval $[0^\circ, 90^\circ)$ that satisfies the equation $\sec\theta = 1.1606249$. Give the answer to six decimal places.

- In fact one such θ would be given by $\theta = \sec^{-1}(1.1606249)$.
- But we don't have an $\sec^{-1}$ Key on our calculator!
- Recall that $\sec\theta = \dfrac{1}{\cos\theta}$, & so $\sec\theta = 1.1606249$ becomes

  $\cos\theta = \dfrac{1}{1.1606249}$

- Now we have $\theta = \cos^{-1}\left(\dfrac{1}{1.1606249}\right) = \boxed{30.502748^\circ}$ ■

8 significant digits

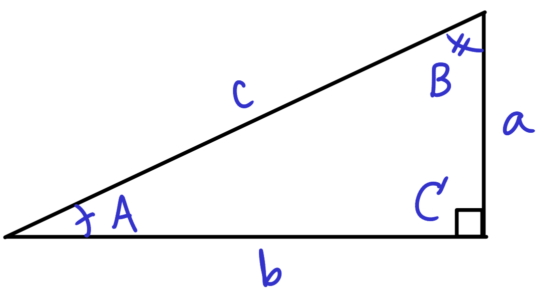
2.4 - Solutions & Applications of Right Triangles

If the angles in a problem are to the nearest...	Example	Write answers to this many significant digits...
Degree	-12°, 103°	2
Ten minutes, or nearest tenth of a degree	$23^\circ 50'$ 58.7°	3
Minute, or hundredth of a degree	$-39^\circ 18'$ 71.69°	4
Ten seconds, or nearest thousandth of a degree	$10^\circ 3' 42''$ 47.301°	5

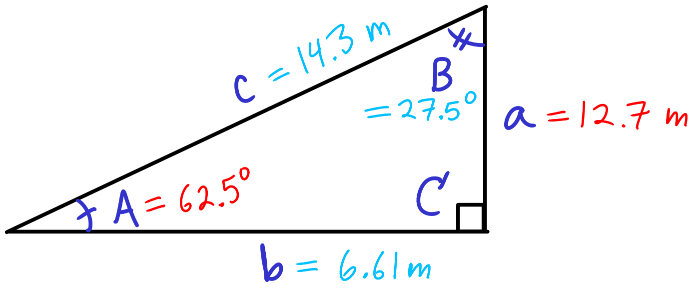
To solve a right triangle means to find the lengths of all its sides and the measures of all its interior angles.

In our book the convention is this:
Side a is opposite angle A
Side b is opposite angle B
Side c is opposite angle C, and C=90° in a right triangle.

So side c is the hypotenuse.



27 Solve the right triangle for which A=62.5° and a=12.7 m.



- Since $A+B+C = 180°$ & $C = 90°$, we have

$$B = 180° - C - A = 180° - 90° - 62.5° = \boxed{27.5°}$$

- Next, to get b we note that $\tan A = \dfrac{\text{opposite side}}{\text{adjacent side}} = \dfrac{a}{b} \Rightarrow$

$$b = \frac{a}{\tan A} = \frac{12.7}{\tan 62.5°} = 6.6112\ldots \approx \boxed{6.61 \text{ m}}$$

- To get c we could use the Pythagorean theorem or use the sine function...

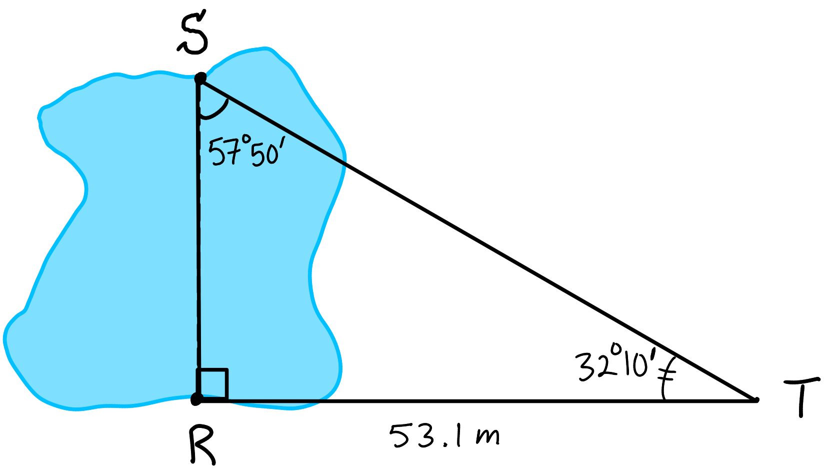
$$c = \sqrt{a^2+b^2} \quad \text{or}$$

$$\sin A = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{a}{c} \Rightarrow c = \frac{a}{\sin A}$$

We get $c = \dfrac{12.7}{\sin 62.5°} = 14.31\ldots = \boxed{14.3 \text{ m}}$

---

46 To find the distance RS across a lake, a surveyor lays off length RT=53.1 m, so that angle T=32°10' and angle S=57°50'. Find length RS.



$$\tan 32°10' = \frac{\text{opposite}}{\text{adjacent}} = \frac{RS}{53.1} \Rightarrow$$

$$RS = 53.1 \tan 32°10' = 53.1 \tan 32.16° = 33.38\ldots \approx \boxed{33.4\ m}$$

$$32°10' = 32° + \left(\frac{10}{60}\right)° = 32.1\overline{6}° = 32.16°$$

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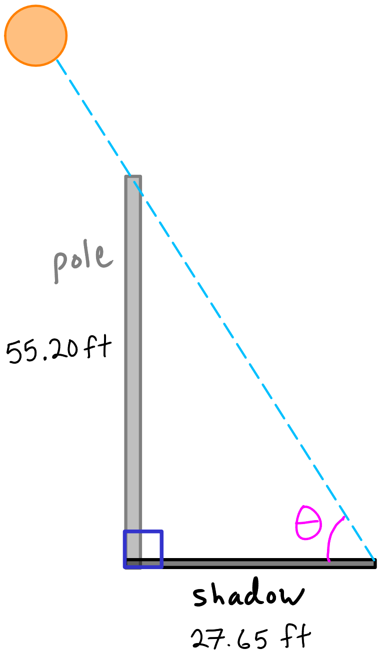
58 The length of the shadow of a flagpole 55.20 ft tall is 27.65 ft. Find the angle of elevation of the sun to the nearest hundredth of a degree.



In the figure the angle of elevation of the sun is denoted by θ.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{55.20}{27.65} = 1.9964,$$

and so...

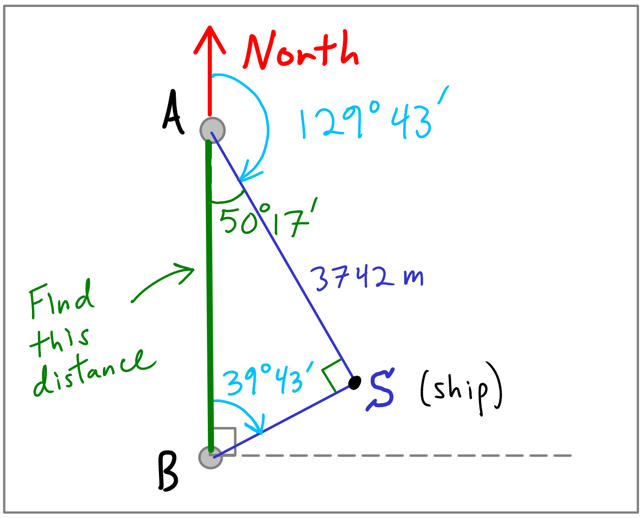
$$\theta = \tan^{-1}(1.9964) = 63.393°\ldots \Rightarrow$$

$$\boxed{\theta = 63.39°}$$

2.5 - Further Applications of Right Triangles

Bearing refers to the direction of motion of an object, or the direction along which an object is sighted.

$\boxed{24}$ Two lighthouses are located on a north-south line. From lighthouse A, the bearing of a ship 3742 m away is 129°43'. From lighthouse B, the bearing of the ship is 39°43'. Find the distance between the lighthouses.



Note that $129°43' - 39°43' = 90°$

Also: $129°43' + A = 180° \Rightarrow$

$A = 180° - 129°43'$

$A = 179°60' - 129°43'$

$A = 50°17' = 50° + \left(\frac{17}{60}\right)^{\circ} = 50.283°$

$A + B + S = 180° \Rightarrow S = 180° - A - B = 180° - 50°17' - 39°43' \Rightarrow$

$S = 180° - (50°17' + 39°43') = 180° - 90° = 90°$ ✓

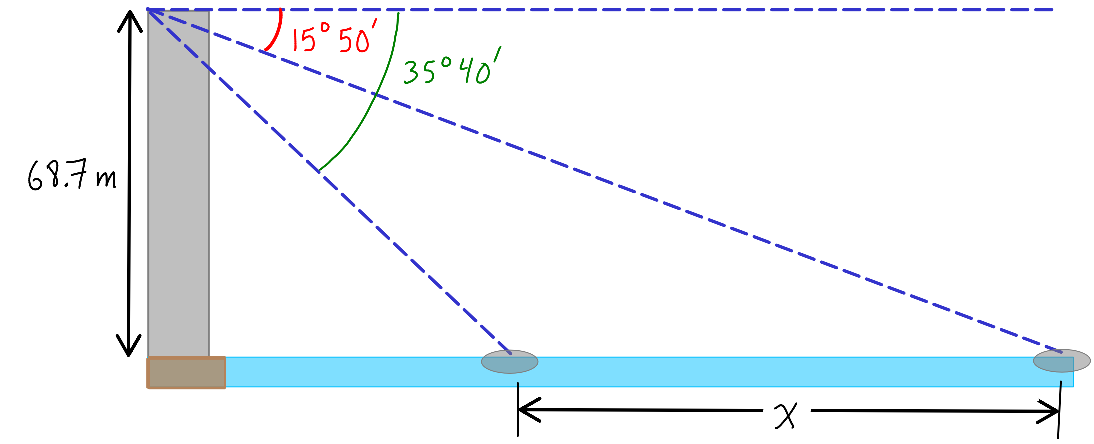
Now, $\cos A = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{AS}{AB} \Rightarrow$

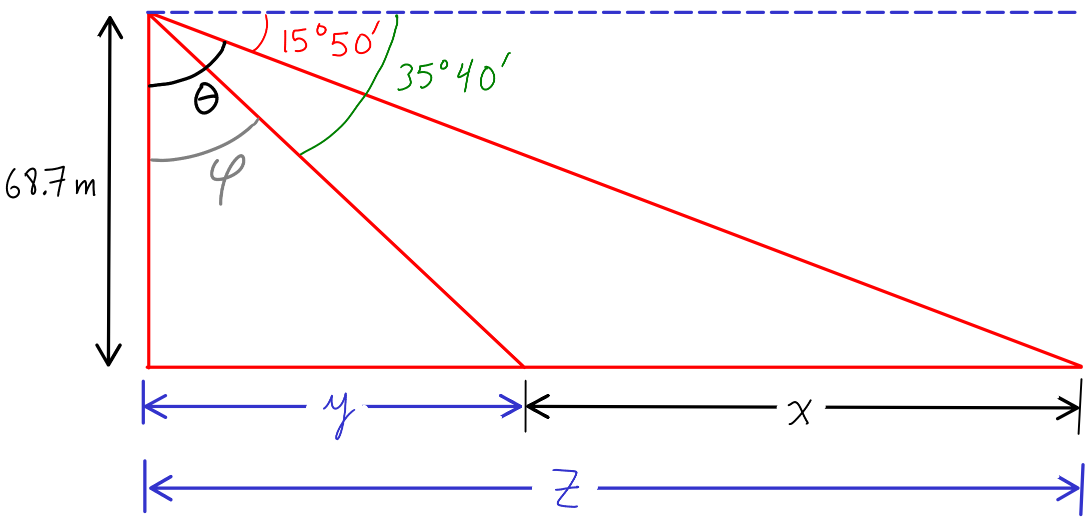
$AB = \dfrac{AS}{\cos A} = \dfrac{3742\text{ m}}{\cos 50.283°} = 5856.06\ldots\text{m}$

$\approx \boxed{5856\text{ m}}$ (4 significant digits)

Exam #1 on Thursday, February 29.

$\boxed{32}$ A whale researcher at the top of a lighthouse is watching a whale approach directly toward the lighthouse. Initially the angle of depression to the whale is 15° 50', and then later, just as the whale turns away from the lighthouse, the angle of depression is 35° 40'. If the height of the lighthouse is 68.7 m, find the distance traveled by the whale as it approached the lighthouse.





- To find the distance x, we find z and y above, and get x = z - y.
- We have $\theta = 90^\circ - 15^\circ 50' = 74^\circ 10' \approx 74.1667^\circ$

  $\varphi = 90^\circ - 35^\circ 40' = 54^\circ 20' \approx 54.3333^\circ$
- Now, $\tan\theta = \dfrac{z}{68.7}$ implies that

  $z = 68.7 \tan\theta = 68.7 \tan 74.1667^\circ = 242.24$ m
- Next, $\tan\varphi = \dfrac{y}{68.7}$ implies that

  $y = 68.7 \tan\varphi = 68.7 \tan 54.3333^\circ = 95.72$ m.

3 significant digits

- So: $x = z - y = 242.24 \text{ m} - 95.72 \text{ m} = 146.52 \text{ m} \approx \boxed{147 \text{ m}}$