

1. 10 pts. A relation R is defined on the set \mathbb{R} of real numbers by $x R y$ if $|x - y| \leq 1$. Which of the properties reflexive, symmetric, and transitive does R possess? Give a proof or disproof in each case.
2. 10 pts. Let $\mathcal{P}(S)$ be the power set of $S = \{a, b, c, d\}$. Define a relation \mathcal{R} on $\mathcal{P}(S)$ as follows: $X \mathcal{R} Y$ if $X \cap Y = \emptyset$. Which of the properties reflexive, symmetric, and transitive does \mathcal{R} possess? Give a proof or disproof in each case.
3. 10 pts. each A relation R is defined on the set \mathbb{R}^+ of positive real numbers by $x R y$ if $\frac{x+y}{2} = \sqrt{xy}$.

 - (a) Prove that R is an equivalence relation.
 - (b) Describe the distinct equivalence classes resulting from R .
4. 10 pts. Let $f : A \rightarrow B$ be a function. Prove that if $A_1, A_2 \subseteq A$, then $f(A_1 \cup A_2) = f(A_1) \cup f(A_2)$.
5. 10 pts. Let $S = \{-2, -1, 0, 1, 2\}$ and let $f = \{(x, y) \in S \times S : |x| + |y| = 2\}$. Prove or disprove that f is a function from S to S .
6. 10 pts. Define $f : \mathbb{R} \rightarrow \mathbb{Z}$ by $f(x) = \lceil x \rceil$ (the ceiling function). Determine, with explanations, whether f is one-to-one or onto.
7. 10 pts. Show that the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 4 - x^5$ is bijective, and determine $f^{-1}(x)$ for $x \in \mathbb{R}$.
8. 10 pts. Let E be the set of even integers and O the set of odd integers. Show that $|E| = |O|$.
9. 10 pts. Let a, b, c , and $n \geq 2$ be integers. Prove that if $a \equiv b \pmod{n}$ and $b \equiv c \pmod{n}$, then $a \equiv c \pmod{n}$.
10. 10 pts. How many different 10-bit strings begin with 1011 or 0110?
11. 10 pts. A license plate consists of a sequence of three letters followed by three digits or three digits followed by three letters. How many different license plates are there?

12. 5 pts. each A total of 36 students plan to take at least one of the courses topology, abstract algebra, and graph theory during the fall. Of these 36 students it is found that 23 plan to take topology, 19 plan to take abstract algebra, 18 plan to take graph theory, 7 plan to take topology and abstract algebra, 9 plan to take topology and graph theory, and 11 plan to take abstract algebra and graph theory.
- (a) How many students plan to take all three courses?
 - (b) How many students plan to take exactly one of the courses?
 - (c) How many students plan to take exactly two of the courses?
13. 10 pts. A bowl contains a large number of red, blue, and yellow marbles. What is the fewest number of marbles that need to be randomly selected from the bowl to be guaranteed that 9 marbles of the same color are chosen?