Math 121
Summer 2024 Name:
Exam 2

1. [5 pts. each] State the negation of the quantified statement.
   (a) For every irrational number x, there exists a rational number y such that $xy = 0$.
   (b) There exists an odd integer m such that for every integer n, mn is odd.

2. [5 pts. each] Consider the following quantified statement: There exist an even integer p and an odd integer q such that $(p+8)^2 + (q-5)^2 = 0$.
   (a) Express the statement in symbols.
   (b) Express the negation of the statement in symbols.
   (c) Express the negation of the statement in words.

3. [10 pts.] Let $a, b \in \mathbb{Z}$. Prove directly that if a and b are odd, then $ab + a + b$ is odd.

4. [10 pts.] Let k, m and n be integers. Prove by contraposition that if $2k + 3m \geq 12n + 1$, then $k \geq 3n + 1$ or $m \geq 2n + 1$.

5. [10 pts.] Let A, B and C be sets. Use a proof by cases to prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

6. [5 pts. each] Prove or disprove each statement.
   (a) Every positive integer can be expressed as the sum of two positive integers.
   (b) For any sets A, B, and C, if $A \cup B = A \cup C$ then $B = C$.

7. [10 pts.] Prove there exist integers m and n of the same parity such that $(m-2)^2 - (n-3)^2 = 1$.

8. [10 pts.] Prove by contradiction that if u and v are positive real numbers, then $\sqrt{u} + \sqrt{v} \neq \sqrt{u+v}$.

9. [10 pts.] Prove by induction: $2 + 5 + 8 + \cdots + (3n-1) = n(3n+1)/2$ for every positive integer n.

10. [10 pts.] Prove by induction: $n^2 > n + 1$ for every integer $n \geq 2$.

11. A sequence is defined recursively by $a_1 = 3$ and $a_n = 2a_{n-1} + 1$ for $n \geq 2$.
   (a) [5 pts.] Determine a_2, a_3, a_4 and a_5.
   (b) [10 pts.] Based on the values of the first several terms of the sequence, guess at a formula for a_n for all $n \in \mathbb{N}$, and use induction to prove that your guess is correct.