

1. 10 pts. For statements  $P$  and  $Q$ , use a truth table to verify that  $\neg(P \vee Q) \not\equiv (\neg P) \vee (\neg Q)$ .

2. 10 pts. Let  $P$ ,  $Q$ , and  $R$  be statements. Use a truth table to determine whether

$$P \wedge (Q \oplus R) \equiv (P \wedge Q) \oplus (P \wedge R).$$

3. 10 pts. For a real number  $x$ , consider the statements  $P : x \leq 2$  and  $Q : x > -4$ . Use De Morgan's Laws to state  $\neg(P \vee Q)$  and  $\neg(P \wedge Q)$ .

4. 10 pts. State in words the converse and the contrapositive of the implication  $P \Rightarrow Q$ , where

$$P : 99 \text{ is even.} \quad Q : 52 \text{ is even.}$$

State the truth value of each of the three implications.

5. 10 pts. Construct the truth table for  $(P \Rightarrow Q) \Rightarrow (\neg P)$ .

6. 5 pts. each Express each of the following implications as an "if-then" sentence.

(a) I will go to the clown circus only if I have the time.

(b) Getting a tax refund is sufficient to make me happy.

7. 5 pts. each For integers  $a$ ,  $b$ ,  $c$  consider the biconditional "At least two of  $a$ ,  $b$  and  $c$  are odd if and only if at least two of  $ab$ ,  $ac$ , and  $bc$  are odd.

(a) Give an example of integers  $a$ ,  $b$ ,  $c$  for which the biconditional is true.

(b) Give an example of integers  $a$ ,  $b$ ,  $c$  for which the biconditional is false.

8. 10 pts. Determine whether  $(P \wedge (\neg Q)) \Rightarrow (P \vee Q)$  is a tautology, contradiction, or neither.

9. 10 pts. Which of the following sets are equal?

$$A = \{n \in \mathbb{Z} : |n| < 2\}, \quad B = \{n \in \mathbb{Z} : n^2 < 4\}, \quad C = \{n \in \mathbb{Z} : n^3 - n = 0\},$$

$$D = \{-1, 0, 1\}, \quad E = \{n \in \mathbb{Z} : n^2 \leq n\}.$$

10. 5 pts. each Determine each set, given that

$$U = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$$

$$A = \{1, 2, 4, 5, 8\}$$

$$B = \{3, 4, 7\}$$

(a)  $\overline{A \cup B}$

(b)  $\overline{A} \cup (A \cap B)$

(c)  $A - \overline{B}$

11. 10 pts. Use a distributive property of logic to verify that

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C).$$

12. 10 pts. For  $A = \{1, 2\}$  and  $B = \{\emptyset\}$ , determine  $\mathcal{P}(A) \times \mathcal{P}(B)$ .

13. 10 pts. List all partitions of the set  $A = \{\alpha, \beta, \gamma, \delta\}$ .