MATH 121
SUMMER 2023 NAME:
EXAM 4

1. $\boxed{\text{5 pts. each}}$ Let $R_1 = \{(a,b) \in \mathbb{R}^2 : a > b\}$, $R_2 = \{(a,b) \in \mathbb{R}^2 : a \geq b\}$, $R_3 = \{(a,b) \in \mathbb{R}^2 : a < b\}$, $R_4 = \{(a,b) \in \mathbb{R}^2 : a \leq b\}$, $R_5 = \{(a,b) \in \mathbb{R}^2 : a = b\}$, $R_6 = \{(a,b) \in \mathbb{R}^2 : a \neq b\}$. Find the following.
   (a) $R_2 - R_1$
   (b) $R_4 \oplus R_6$
   (c) $R_1 \circ R_5$
   (d) $R_3 \circ R_3$

2. $\boxed{\text{10 pts.}}$ Let R be a relation that is reflexive and transitive. Prove that $R^n = R$ for all $n \in \mathbb{Z}^+$.

3. $\boxed{\text{5 pts. each}}$ Let R_1 and R_2 be relations on a set A represented by the matrices

$$\mathbf{M}_{R_1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{M}_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

   Find the matrices that represent
   (a) $R_2 \circ R_1$
   (b) $R_1 \oplus R_2$

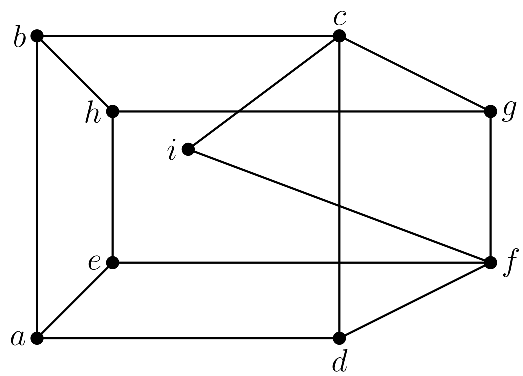
4. $\boxed{\text{10 pts.}}$ Suppose $A \neq \varnothing$ and R is an equivalence relation on A. Show that there is a function f with domain A such that $(x,y) \in R$ if and only if $f(x) = f(y)$.

5. $\boxed{\text{5 pts. each}}$ What is the congruence class of $[n]_6$ (that is, the equivalence class of n with respect to congruence modulo 6) when
   (a) $n = 4$
   (b) $n = -5$

6. $\boxed{\text{10 pts.}}$ Let $G = (V,E)$ be a simple graph. Show that the relation R on V defined so that uRv if and only if there is an edge associated with $\{u,v\}$ is a symmetric, irreflexive relation on V. (A relation on a set is **irreflexive** if no element of the set is related to itself.)

7. $\boxed{\text{10 pts.}}$ Show that in a simple graph with at least two vertices there must be two vertices that have the same degree.

8. $\boxed{\text{10 pts.}}$ Determine whether the graph below is bipartite. If it is, then give a bipartition (V_1, V_2).



9. $\boxed{\text{10 pts.}}$ The **converse** of a directed graph $G = (V, E)$, denoted by G^c, is the directed graph (V, F), where the set F is obtained by reversing the direction of each edge in E. Show that $G^c = G$ if and only if the relation associated with G is symmetric. (Recall a directed graph has no multiple edges.)

10. $\boxed{\text{10 pts.}}$ Draw a graph with adjacency matrix

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}.$$

11. $\boxed{\text{10 pts.}}$ Draw an undirected graph represented by the adjacency matrix

$$\begin{bmatrix} 1 & 3 & 2 \\ 3 & 0 & 4 \\ 2 & 4 & 0 \end{bmatrix}.$$