NAME:

1. [10 pts.] Use induction to prove that, for all positive integers $n$,

$$\sum_{k=1}^{n} k2^k = (n-1)2^{n+1} + 2.$$

2. [10 pts.] Use induction to prove that 3 divides $n^3 + 2n$ whenever $n$ is a positive integer.

3. [10 pts.] Let $a_0 = 1$, $a_1 = 2$, and $a_n = 2a_{n-1} + a_{n-2}$ for $n \geq 2$. Prove that $a_n \leq \left(\frac{5}{2}\right)^n$ for $n \geq 0$.

4. [10 pts.] How many different functions are there from the set $\{a, b, c, d, e, f, g, h, i, j\}$ to the set $\{2, 4, 6, 8\}$?

5. [10 pts.] How many integers in the set $\{1000, 1001, 1002, \ldots, 9999\}$ are not divisible by 3 or 13?

6. [10 pts.] Let $d$ be a positive integer. Use the Pigeonhole Principle to show that among any collection of $d+1$ integers there are two with exactly the same remainder when they are divided by $d$. The integers are not necessarily consecutive!

7. [10 pts.] How many permutations of the letters ABCDEFGHI contain the strings $AB$ and $FGI$?

8. [10 pts.] How many ways are there for three ferrets and six gerbils to stand in line so that all ferrets stand together?

9. [10 pts.] How many ways are there to select 12 countries in the United Nations to serve on a council if 3 are selected from a block of 45, 4 from a block of 57, and the rest from the remaining 69 countries?

10. [10 pts.] What is the probability that a five-card poker hand contains cards of five different kinds?

11. [10 pts.] What is the probability that a positive integer not exceeding 3500 selected at random is divisible by 7 or 11?

12. [10 pts.] In a survey of 270 pointy-headed professors, it is found that 64 like parsnips, 94 like carrots, 58 like radishes, 26 like both parsnips and carrots, 28 like both parsnips and radishes, 22 like both carrots and radishes, and 14 like all three root vegetables. How many of the 270 professors do not like any of these vegetables?