

1. 10 pts. Prove or disprove: If  $A$ ,  $B$ , and  $C$  are nonempty sets such that  $A \times B = A \times C$ , then  $B = C$ .
2. 10 pts. Let  $A = \{b, c, d, e\}$  and  $B = \{a, c, d, e, f\}$ . Find  $A \cup B$ ,  $A \cap B$ ,  $A - B$ , and  $B - A$ .
3. Let  $A$ ,  $B$ , and  $C$  be sets. Prove the following (do not use Venn diagrams):

  - (a) 5 pts.  $A \cap (B - A) = \emptyset$
  - (b) 10 pts.  $(B - A) \cup (C - A) = (B \cup C) - A$
4. 10 pts. Let  $f : \mathbb{R} \rightarrow (0, \infty)$ . Show that  $f(x)$  is strictly increasing if and only if the function  $g(x) = 1/f(x)$  is strictly decreasing. How do we know that  $g(x)$  is defined for all  $x \in \mathbb{R}$ ?
5. 5 pts. each Let  $S = \{-1, 0, 2, 4, 7\}$ . Find  $f(S)$  if:

  - (a)  $f(x) = -3x + 5$
  - (b)  $f(x) = \lfloor (x^2 + 1)/4 \rfloor$
6. 10 pts. Using an iterative approach, find the solution to the recurrence relation  $a_n = a_{n-1} - n$ , where  $a_0 = 4$ .
7. 5 pts. Compute the double sum:

$$\sum_{i=1}^3 \sum_{j=0}^3 (2i - 3j).$$
8. 10 pts. Find the product  $AB$ , where

$$A = \begin{bmatrix} -1 & a \\ b & 0 \\ 2 & -3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & c & 1 \\ -5 & a & b \end{bmatrix}.$$
9. 5 pts. What time does a 24-hour clock read 1013 hours after it reads 17:00? Show whatever work you did to arrive at the answer.

10. 10 pts. Suppose  $a, b \in \mathbb{Z}$ ,  $a \equiv 10 \pmod{17}$ , and  $b \equiv 7 \pmod{17}$ . Find the integer  $c \in [0, 16]$  such that  $c \equiv a - 4b \pmod{17}$ .
11. 5 pts. each Convert:
- (a)  $(FAD09)_{16}$  to its binary expansion.
  - (b) 643 to its binary expansion.
12. 10 pts. Find the product:  $(7301)_8 \cdot (54)_8$ . Express the answer as an octal expansion.
13. 10 pts. each Show the process in doing each of the following.
- (a) Find the prime factorization for 6174 and 7800.
  - (b) Find  $\gcd(6174, 7800)$  and  $\text{lcm}(6174, 7800)$ .