

MATH 121
SUMMER 2023
EXAM 1

NAME:

1. 5 pts. each Let p : “I bought a lottery ticket this week,” and let q : “I won the million dollar jackpot.” Express each of the following as an English sentence.
 - (a) $\neg p \rightarrow \neg q$
 - (b) $\neg p \vee (p \wedge q)$

2. 5 pts. each Let p : “You get an A on the final exam,” let q : “You do every exercise in this book,” and let r : “You get an A in this class.” Express the following using p , q , r , and logical operators.
 - (a) You get an A in this class if and only if you either do every exercise in this book or you get an A on the final.
 - (b) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.

3. 10 pts. Construct a truth table for the compound proposition $(p \rightarrow q) \oplus (p \leftrightarrow \neg q)$. There are four operations to execute, so, using the proper order of operations, display the truth values that result from each operation as a column under that operation. Circle the answer column.

4. 10 pts. Are these system specifications consistent? “Whenever the server is down, browsers cannot view the website. If browsers can view the website, then they can enter the forum. If browsers can’t enter the forum, then the server is not down.”

5. 10 pts. Use truth tables to show that $p \leftrightarrow q$ and $\neg p \leftrightarrow \neg q$ are logically equivalent. Show all columns of truth values that lead to the answer columns, and circle the answer columns.

6. 5 pts. each Let $R(x)$ be “ x is a rabbit,” and let $H(x)$ be “ x hops.” The domain consists of all animals. Translate into English:
 - (a) $\forall x(R(x) \rightarrow H(x))$
 - (b) $\exists x(\neg R(x) \wedge H(x))$

7. 5 pts. each Translate each of the following statements into logical expressions using predicates, quantifiers, and logical operators:
 - (a) All tools are in the correct place and are in excellent condition.
 - (b) None of your tools is in the correct place, but some of them are in excellent condition.

8. 5 pts. each Let $I(x)$ be “ x has an Internet connection” and $T(x, y)$ be “ x and y have talked over the Internet.” The domain for all variables consists of all students in your class. Use quantifiers to express each of the following.
- (a) Not everyone in your class has an Internet connection.
 - (b) Everyone except one student in your class has an Internet connection.
 - (c) Someone in your class has an Internet connection but has not talked over the Internet with anyone else in your class.
9. 5 pts. State the rule of inference used in this argument: “It is either hotter than 100 degrees today or the pollution is dangerous. It is less than 100 degrees outside today. Therefore, the pollution is dangerous.”
10. 10 pts. Use a direct proof to show that the product of two rational numbers is rational.
11. 10 pts. Prove or disprove that the product of a nonzero rational number and an irrational number is irrational.