Math 121 Exam #1 Key (Summer 2024)

1 The truth values of the statements only match in cases 2 and 3:

P	Q	$\neg(P \vee Q)$	$(\neg P) \vee (\neg Q)$
1	1	0	0
1	0	0	1
0	1	0	1
0	0	1	1

2 The truth values match in all cases, so $P \wedge (Q \oplus R) \equiv (P \wedge Q) \oplus (P \wedge R)$.

P	Q	R	$P \wedge (Q \oplus R)$	$(P \wedge Q) \oplus (P \wedge R)$
1	1	1	0	0
1	1	0	1	1
1	0	1	1	1
1	0	0	0	0
0	1	1	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	0	0

3 $\neg(P \vee Q) \equiv (\neg P \wedge \neg Q)$: "$x > 2$ and $x \leq -4$."
$\neg(P \wedge Q) \equiv (\neg P \vee \neg Q)$: "$x > 2$ or $x \leq -4$."

4 Converse is $Q \Rightarrow P$: "If 52 is even, then 99 is even." Contrapositive is $\neg Q \Rightarrow \neg P$: "If 52 is not even, then 99 is not even." $P \Rightarrow Q$ is true, $Q \Rightarrow P$ is false, $\neg Q \Rightarrow \neg P$ is true.

5

P	Q	$(P \Rightarrow Q) \Rightarrow \neg P$
1	1	0
1	0	1
0	1	1
0	0	1

6a "If I will go to the clown circus, then I have the time."

6b "If I get a tax refund, then I am happy." (Note that the original statement needs several modification to create a grammatically correct English if-then statement.)

7a $a = b = c = 1$.

7b $a = b = 1$ and $c = 2$.

8 The statement is a tautology.

P	Q	$(P \wedge (\neg Q)) \Rightarrow (P \vee Q)$
1	1	1
1	0	1
0	1	1
0	0	1

9 $A = \{-1, 0, 1\}$, $B = \{-1, 0, 1\}$, $C = \{-1, 0, 1\}$, $D = \{-1, 0, 1\}$, $E = \{0, 1\}$. Thus $A = B = C = D$.

10a $\overline{A \cup B} = \{0, 6\}$

10b $\overline{A} \cup (A \cap B) = \{0, 3, 6, 7\} \cup \{4\} = \{0, 3, 4, 6, 7\}$

10c $A - \overline{B} = \{1, 2, 4, 5, 8\} - \{0, 1, 2, 5, 6, 8\} = \{4\}$

11 We use the property $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$:

$$\begin{aligned} x \in A \cap (B \cup C) &\equiv (x \in A) \wedge (x \in B \cup C) \\ &\equiv (x \in A) \wedge ((x \in B) \vee (x \in C)) \\ &\equiv ((x \in A) \wedge (x \in B)) \vee ((x \in A) \wedge (x \in C)) \\ &\equiv (x \in A \cap B) \vee (x \in A \cap C) \\ &\equiv x \in (A \cap B) \cup (A \cap C) \end{aligned}$$

12 $\mathcal{P}(A) = \{\varnothing, \{1\}, \{2\}, A\}$ and $\mathcal{P}(B) = \{\varnothing, B\}$, so we have

$$\mathcal{P}(A) \times \mathcal{P}(B) = \{(\varnothing, \varnothing), (\varnothing, B), (\{1\}, \varnothing), (\{1\}, B), (\{2\}, \varnothing), (\{2\}, B), (A, \varnothing), (A, B)\}.$$

13 There are 15 partitions in all...

$$\begin{aligned} &\{A\},\ \ \{\{\alpha\}, \{\beta\}, \{\gamma\}, \{\delta\}\}, \\ &\{\{\alpha, \beta\}, \{\gamma\}, \{\delta\}\},\ \ \{\{\alpha, \gamma\}, \{\beta\}, \{\delta\}\},\ \ \{\{\alpha, \delta\}, \{\beta\}, \{\gamma\}\}, \\ &\{\{\beta, \gamma\}, \{\alpha\}, \{\delta\}\},\ \ \{\{\beta, \delta\}, \{\alpha\}, \{\gamma\}\},\ \ \{\{\gamma, \delta\}, \{\alpha\}, \{\beta\}\}, \\ &\{\{\alpha, \beta\}, \{\gamma, \delta\}\},\ \ \{\{\alpha, \gamma\}, \{\beta, \delta\}\},\ \ \{\{\alpha, \delta\}, \{\beta, \gamma\}\}, \\ &\{\{\alpha, \beta, \gamma\}, \{\delta\}\},\ \ \{\{\alpha, \beta, \delta\}, \{\gamma\}\},\ \ \{\{\alpha, \gamma, \delta\}, \{\beta\}\},\ \ \{\{\beta, \gamma, \delta\}, \{\alpha\}\}. \end{aligned}$$