Math 121 Exam #1 Key (Summer 2023)

1a If I did not buy a lottery ticket this week, then I did not win the million dollar jackpot.

1b I did not buy a lottery ticket this week, or I bought a lottery ticket this week and I won the million dollar jackpot.

2a $r \leftrightarrow (q \vee p)$

2b $(p \wedge q) \to r$

3 The answer column is in red.

p	q	$(p \to q)$	$\oplus$	$(p$	$\leftrightarrow$	$\neg q)$
1	1	1	1		0	0
1	0	0	1		1	1
0	1	1	0		1	0
0	0	1	1		0	1

4 Let p: The server is down, q: Browsers can view the website, r: Browsers can enter the forum. The specifications are $p \to \neg q$, $q \to r$, $\neg r \to \neg p$. All these specifications can be made simultaneously true if we assume, for instance, that p is false, q is false, and r is true. There may be other possibilities.

5 I'm just showing the answer columns here. Since the truth values match in all four cases, the statements are equivalent.

p	q	$p \leftrightarrow q$	$\neg p \to \neg q$
1	1	1	1
1	0	0	0
0	1	0	0
0	0	1	1

6a Any animal that is a rabbit hops.

6b There is an animal that hops but is not a rabbit. *Or:* There is an animal that is not a rabbit and hops.

7a Let $C(x) =$"x is in the correct place" and $E(x) =$"x is in excellent condition." Domain for x will be all tools. Then: $\forall x(C(x) \wedge E(x))$.

7b Like part (a), only now the domain for x will be *your* tools. Then: $\forall x(\neg C(x)) \wedge \exists x E(x)$, or equivalently $\neg \exists x C(x) \wedge \exists x E(x)$

8a $\neg \forall x I(x)$ or $\exists x(\neg I(x))$.

8b $\exists x[\neg I(x) \wedge \forall y(\neg I(y) \to x = y)]$

8c $\exists x[I(x) \land \forall y(x \neq y \rightarrow \neg T(x,y))]$

9 Disjunctive syllogism

10 Let a and b be rational numbers. Then there exist integers p, $q \neq 0$, r, $s \neq 0$ such that $a = p/q$ and $b = r/s$. Now,

$$ab = \frac{p}{q} \cdot \frac{r}{s} = \frac{pr}{qs},$$

and since pr and qs are integers, with $qs \neq 0$, we conclude that ab is a rational number.

11 Let x be irrational, and let $r \neq 0$ be rational, so that $r = m/n$ for integers $m, n \neq 0$. Suppose rx is rational. Then $rx = p/q$ for integers p and q, with $q \neq 0$. Now,

$$x = \frac{p}{rq} = \frac{np}{mq},$$

where np and $mq \neq 0$ are integers, and so x is rational. This is a contradiction, and therefore rx must be irrational.