Math 120 Exam #2 Key (Winter 2015)

1a $5x^2 - 3x - 2 = 0 \;\Rightarrow\; (5x+2)(x-1) = 0 \;\Rightarrow\; 5x+2 = 0$ or $x - 1 = 0$, so $x = 1, -2/5$

1b $x^2 - 3x = 6 \;\Rightarrow\; x^2 - 3x + 9/4 = 6 + 9/4 \;\Rightarrow\; \left(x - \frac{3}{2}\right)^2 = \frac{33}{4} \;\Rightarrow\; x - \frac{3}{2} = \pm\frac{\sqrt{33}}{2} \;\Rightarrow$
$x = \frac{3}{2} \pm \frac{\sqrt{33}}{2}$

2 If w is the width of the metal sheet, then the length of the sheet is $w + 10$. However, the *box* has width $w - 4$ and length $(w+10) - 4 = w + 6$, and the height must be 2. The volume V of the box is computed as $V = 2(w-4)(w+6)$, but we're also given that $V = 832$. This gives us an equation: $2(w-4)(w+6) = 832$. Hence $w^2 + 2w - 440 = 0$, which leads to $(w+22)(w-20) = 0$ and so $w = -22, 20$. Clearly the width of the original sheet can't be -22 cm, which leaves it to be 20 cm. Dimensions of sheet: 20 cm by 30 cm.

3 Let w be the width, so the length is $w + 6$. By the Pythagorean Theorem the length of the diagonal is $\sqrt{w^2 + (w+6)^2}$, which is given to be 174, so that

$$w^2 + (w+6)^2 = 174^2 \;\Rightarrow\; w^2 + 6w - 15{,}120 = 0 \;\Rightarrow\; (w-120)(w-126) = 0.$$

Thus the width is 120, and then the length is 126. Dimensions are 126 m $\times$ 120 m.

4

	Rate of Work	Time Worked	Fraction of Job Done
Emperor	$\frac{1}{280}$	t	$\frac{t}{280}$
Vader	$\frac{1}{700}$	t	$\frac{t}{700}$

Let t be the time it would take to complete the job. We get

$$\frac{t}{280} + \frac{t}{700} = 1 \;\Rightarrow\; 700t + 280t = (700)(280) \;\Rightarrow\; 980t = 196{,}000 \;\Rightarrow\; t = 200 \text{ hours.}$$

5a Multiply both sides of the equation by $(x-2)(x+2)$:

$$(x-2)(x+2)\left(\frac{x+5}{x-2} = \frac{5}{x+2} + \frac{28}{(x-2)(x+2)}\right) \;\Rightarrow\; (x+2)(x+5) = 5(x-2) + 28.$$

Thus we have $x^2 + 7x + 10 = 5x + 18$, and then $(x+4)(x-2) = 0$, and finally $x = -4, 2$. However, 2 is an extraneous solution, so the solution set is $\{-4\}$.

5b $\sqrt{2x} = x - 4 \;\Rightarrow\; 2x = (x-4)^2 \;\Rightarrow\; x^2 - 10x + 16 = 0 \;\Rightarrow\; (x-8)(x-2) = 0 \;\Rightarrow$ $x = 2, 8$. But 2 is extraneous (it gives us $2 = -2$ in the original equation), so solution set is $\{8\}$.

5c $\sqrt{x} = \sqrt{x+3} - 1 \;\Rightarrow\; x = (\sqrt{x+3} - 1)^2 \;\Rightarrow\; x = (x+3) - 2\sqrt{x+3} + 1 \;\Rightarrow\; 2\sqrt{x+3} = 4 \;\Rightarrow\; 4(x+3) = 16 \;\Rightarrow\; x = 1$. Solution set: $\{1\}$.

5d Let $u = x^2$, so equation becomes $u^2 - 5u + 4 = 0$, which becomes $(u-1)(u-4) = 0$ and gives $u = 1, 4$. Now, $x^2 = 1$ yields $x = \pm 1$, and $x^2 = 4$ yields $x = \pm 2$. Solution set: $\{-1, 1, -2, 2\}$.

5e $|8-2x| = 42$ implies that $8 - 2x = \pm 42$. Now, $8 - 2x = 42$ gives $x = -17$, and $8 - 2x = -42$ gives $x = 25$. Solution set: $\{-17, 25\}$.

6a We get $4x + 3 \geq 3x + 5$, and thus $x \geq 2$. Solution set is $[2, \infty)$.

6b $-18 < x - 4 < 12 \;\Rightarrow\; -14 < x < 16$, so solution set is $(-14, 16)$.

6c $6x^2 - 11x - 10 < 0 \;\Rightarrow\; (3x+2)(2x-5) < 0$. Case 1: $3x + 2 < 0$ & $2x - 5 > 0$, which leads to a contradiction. Case 2: $3x + 2 > 0$ & $2x - 5 < 0$, which leads to $-\frac{2}{3} < x < \frac{5}{2}$. Solution set: $\left(-\frac{2}{3}, \frac{5}{2}\right)$.

6d $2x^3 - 3x^2 - 5x \leq 0 \;\Rightarrow\; x(2x-5)(x+1) \leq 0$. Case 1: $x \leq 0$, $2x - 5 \geq 0$, $x + 1 \geq 0$, which leads to contradiction. Case 2: $x \geq 0$, $2x - 5 \leq 0$, $x + 1 \geq 0$, which leads to $0 \leq x \leq \frac{5}{2}$. Case 3: $x \geq 0$, $2x - 5 \geq 0$, $x + 1 \leq 0$, again contradictory. Case 4: $x \leq 0$, $2x - 5 \leq 0$, $x + 1 \leq 0$, which leads to $x \leq -1$. Solution set: $(-\infty, -1] \cup \left[0, \frac{5}{2}\right]$.

6e We have

$$\frac{10}{2x-3} \leq 5 \;\Rightarrow\; \frac{10}{2x-3} - \frac{5(2x-3)}{2x-3} \leq 0 \;\Rightarrow\; \frac{25-10x}{2x-3} \leq 0.$$

Case 1: $25 - 10x \leq 0$ & $2x - 3 > 0$, which yields $x \geq \frac{5}{2}$ & $x > \frac{3}{2}$, and therefore $x \geq \frac{5}{2}$.
Case 2: $25 - 10x \geq 0$ & $2x - 3 < 0$, which yields $x \leq \frac{5}{2}$ & $x < \frac{3}{2}$, and therefore $x \geq \frac{3}{2}$.
Solution set: $\left(-\infty, \frac{3}{2}\right) \cup \left[\frac{5}{2}, \infty\right)$.

6f $|8x - 3| > 13$ implies $8x - 3 < -13$ or $8x - 3 > 13$, and hence either $x < -5/4$ or $x > 2$. Solution set: $\left(-\infty, -\frac{5}{4}\right) \cup (2, \infty)$.

7 $\sqrt{(-6-8)^2 + (5-(-2))^2} = \sqrt{245} = 7\sqrt{5}$.

8 $(x^2 + 8x + 16) + (y^2 - 6y + 9) = -16 + 16 + 9 \;\Rightarrow\; (x+4)^2 + (y-3)^2 = 3^2$, which is a circle with center at $(-4, 3)$ and radius 3.