Math 120 Exam #4 Key (Winter 2013)

1 We have

$$\begin{array}{r|rrrrr|r} -3 & 1 & -1 & 2 & 0 & 3 & 1 \\ & & -3 & 12 & -42 & 126 & -387 \\ \hline & 1 & -4 & 14 & -42 & 129 & -386 \end{array} \quad \longrightarrow \quad x^4 - 4x^3 + 14x^2 - 42x + 129 - \frac{386}{x+3}.$$

2a The candidates are of the form (factor of 4)/(factor of 2):

$$\frac{\pm 1, \pm 2, \pm 4}{\pm 1, \pm 2} = \pm 1, \pm 2, \pm 4, \pm \frac{1}{2}.$$

2b The division

$$\begin{array}{r|rrrr|r} -2 & 2 & 15 & 31 & 20 & 4 \\ & & -4 & -22 & -18 & -4 \\ \hline & 2 & 11 & 9 & 2 & 0 \end{array}$$

followed by

$$\begin{array}{r|rrr|r} -1/2 & 2 & 11 & 9 & 2 \\ & & -1 & -5 & -2 \\ \hline & 2 & 10 & 4 & 0 \end{array}$$

show that -2 and $-1/2$ are zeros for the function f, and we obtain the factorization

$$f(x) = (x+2)(x+1/2)(2x^2+10x+4) = (x+2)(2x-1)(x^2+5x+2)$$

from the bottom row of numbers in the second division. Now $f(x) = 0$ implies (by the Zero Factor Principle from way back) that $x + 2 = 0$ or $2x - 1 = 0$ or $x^2 + 5x + 2 = 0$. The last equation yields two additional zeros: $(-5 \pm \sqrt{17})/2$.

So the zeros of f are: -2, $-\frac{1}{2}$, $-\frac{5}{2} + \frac{\sqrt{17}}{2}$, $-\frac{5}{2} - \frac{\sqrt{17}}{2}$.

2c By the Factor Theorem $f(x)$ factors as

$$f(x) = (x+2)(2x+1)\left[x - \left(-\tfrac{5}{2} + \tfrac{\sqrt{17}}{2}\right)\right]\left[x - \left(-\tfrac{5}{2} - \tfrac{\sqrt{17}}{2}\right)\right].$$

3 By the Factor Theorem we must have $f(x) = c(x+2)(x-1)(x-3)$ for some appropriate constant c. From this we see that

$$f(0) = c(0+2)(0-1)(0-3) = 6c,$$

and since we're given that $f(0) = 8$, it follows that $6c = 8$ and thus $c = \frac{4}{3}$. Therefore $f(x) = \frac{4}{3}(x+2)(x-1)(x-3)$, or equivalently

$$f(x) = \tfrac{4}{3}x^3 - \tfrac{8}{3}x^2 - \tfrac{20}{3}x + 8.$$

4 By the Conjugate Zeros Theorem $2+i$ must also be a zero in order for $f(x)$ to have real coefficients. Thus by the Factor Theorem we have

$$f(x) = (x-1)[x-(2-i)][x-(2+i)] = (x-1)(x^2-4x+5) = x^3-5x^2+9x-5.$$

5a Equate bases: $(4^3)^{2x-1} = 4^{3x} \;\Rightarrow\; 4^{6x-3} = 4^{3x} \;\Rightarrow\; 6x-3 = 3x \;\Rightarrow\; x = 1.$

5b Convert to exponential equation: $8^y = \sqrt[4]{8} \;\Rightarrow\; 8^y = 8^{1/4} \;\Rightarrow\; y = 1/4.$

5c Convert to exponential equation: $x^{-2} = 3 \;\Rightarrow\; x^2 = 1/3 \;\Rightarrow\; x = \pm 1/\sqrt{3}$; but the base of a logarithm cannot be negative, so the only solution is $x = 1/\sqrt{3}$.

6a Taking the natural logarithm of both sides: $6^{x+3} = 8^x \;\Rightarrow\; \ln(6^{x+3}) = \ln(8^x) \;\Rightarrow\; (x+3)\ln 6 = x\ln 8 \;\Rightarrow\; x\ln 8 - x\ln 6 = 3\ln 6 \;\Rightarrow\; x = \dfrac{3\ln 6}{\ln 8 - \ln 6} \approx 18.6848.$

6b Since any logarithmic function is one-to-one, we obtain $3x+8 = 32$ and hence $x = 8$.

6c $\log_2 x + \log_2(x+2) = 3 \;\Rightarrow\; \log_2[x(x+2)] = 3 \;\Rightarrow\; 2^3 = x(x+2) \;\Rightarrow\; x^2+2x-8 = 0 \;\Rightarrow\; (x+4)(x-2) = 0 \;\Rightarrow\; x = -4, 2$. But -4 is extraneous since it results in the logarithm of a negative number in the original equation, and so the only solution is $x = 2$.

7 Using the appropriate formula, we have

$$8000 = 5000\left(1+\frac{0.07}{4}\right)^{4t} \;\Rightarrow\; 1.0175^{4t} = 1.6 \;\Rightarrow\; \ln(1.0175^{4t}) = \ln 1.6,$$

and so

$$4t\ln 1.0175 = \ln 1.6 \;\Rightarrow\; t = \frac{\ln 1.6}{4\ln 1.0175} \approx 6.7729.$$

It will take about 6.8 years.

8 Whatever the starting amount P is, we want to find the time t at which $A = 2P$. Using the appropriate formula,

$$2P = Pe^{0.042t} \;\Rightarrow\; e^{0.042t} = 2 \;\Rightarrow\; 0.042t = \ln 2 \;\Rightarrow\; t = \frac{\ln 2}{0.042} \approx 16.504.$$

It will take about 16.5 years.

9a $n(4) = 12e^{0.012(4)} \approx 12.59$, or $12,590,000$.

9b Find t for which $n(t) = 35$, which results in the equation $12e^{0.012t} = 35$. Solving:

$$e^{0.012t} = 35/12 \quad \Rightarrow \quad 0.012t = \ln(35/12) \quad \Rightarrow \quad t = \frac{\ln(35/12)}{0.012} \approx 89.2.$$

It will take about 89.2 years.

10a Multiply the second equation by -4 to get

$$\begin{cases} 12x - 5y = 9 \\ -12x + 32y = 72 \end{cases}$$

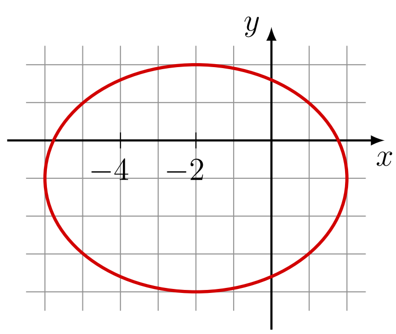
Adding the equations then yields $27y = 81$, or $y = 3$. Substitution this into the first equation to get $12x - 5(3) = 9$, or $x = 2$. Solution is $(2, 3)$.

10b The first equation yields $z = 2x + 6y - 6$. Substituting this into the second and third equations yields

$$\begin{cases} 14x + 27y = 25 \\ 2x - 3y = -1 \end{cases}$$

This system is satisfied when $x = 1/2$ and $y = 2/3$, and so the solution to the original system is $\left(\frac{1}{2}, \frac{2}{3}, -1\right)$.

11 Graph:



Domain: $[-6, 2]$
Range: $[-4, 2]$
Center: $(-2, -1)$
Vertices: $(-6, -1)$, $(2, -1)$, $(-2, -4)$, $(-2, 2)$
Foci: $(-2 - \sqrt{7}, -1)$, $(-2 + \sqrt{7}, -1)$