# MATH 120 EXAM #3 KEY (WINTER 2013)

**1** Solve $3x - 4y = 2$ for $y$ to get $y = \frac{3}{4}x - \frac{1}{2}$, which shows the slope for the line given by $3x - 4y = 2$ to be $\frac{3}{4}$. Since the line $L$ through $(-2, -7)$ is parallel to this line, its slope must also be $\frac{3}{4}$. So the equation for $L$ is $y - (-7) = \frac{3}{4}(x - (-2))$, or $y = \frac{3}{4}x - \frac{11}{2}$.

**2** Solve $8x - 3y = 6$ for $y$ to get $y = \frac{8}{3}x - 2$, which shows the slope for the line given by $8x - 3y = 6$ to be $\frac{8}{3}$. Since the line $L$ through $(2, -4)$ is perpendicular to this line, its slope must also be $-\frac{3}{8}$. So the equation for $L$ is $y - (-4) = -\frac{3}{8}(x - 2)$, or $y = -\frac{3}{8}x - \frac{13}{4}$.

**3** Domain is $[-3, 3]$ and range is $[-4, 4]$. Relation is not a function since it contains two distinct ordered pairs having the same first component value: $(0, 4)$ and $(0, -4)$.

**4** $f(-8) = (-8)^2 + \sqrt[3]{-8} = 64 + (-2) = 62$ and $f(c) = c^2 + \sqrt[3]{c}$

**5a** Domain and range are both $(-\infty, \infty)$

**5b** Domain is $(-\infty, \infty)$ and range is $[25, \infty)$

**6a** $\text{Dom}(\alpha) = \left\{x \mid x \neq \frac{2}{3}\right\} = \left(-\infty, \frac{2}{3}\right) \cup \left(\frac{2}{3}, \infty\right)$

**6b** $\text{Dom}(\beta) = \{x \mid 9x - 5 \geq 0\} = \left\{x \mid x \geq \frac{5}{9}\right\} = \left[\frac{5}{9}, \infty\right)$

**6c** $\text{Dom}(\gamma) = \{x \mid 36 - x^2 \geq 0\} = \{x \mid x^2 \leq 36\} = \{x \mid -6 \leq x \leq 6\} = [-6, 6]$

**7a** By definition we have $(\alpha + \gamma)(x) = \alpha(x) + \gamma(x) = \dfrac{x+1}{3x-2} + \sqrt{36 - x^2}$. Also

$$\text{Dom}(\alpha + \gamma) = \text{Dom}(\alpha) \cap \text{Dom}(\gamma) = \left[\left(-\infty, \tfrac{2}{3}\right) \cup \left(\tfrac{2}{3}, \infty\right)\right] \cap [-6, 6] = \left[-6, \tfrac{2}{3}\right) \cup \left(\tfrac{2}{3}, 6\right].$$

**7b** We find $(\alpha/\beta)(x) = \alpha(x)/\beta(x) = \dfrac{x+1}{3x-2} \cdot \dfrac{1}{\sqrt{9x-5}} = \dfrac{x+1}{(3x-2)\sqrt{9x-5}}$, and

$$\begin{aligned}
\text{Dom}(\alpha/\beta) &= \{x \mid x \in \text{Dom}(\alpha) \cap \text{Dom}(\beta) \ \ \& \ \ \beta(x) \neq 0\} \\
&= \left\{x \mid x \in \left(-\infty, \tfrac{2}{3}\right) \cup \left(\tfrac{2}{3}, \infty\right) \ \ \& \ \ x \in \left[\tfrac{5}{9}, \infty\right) \ \ \& \ \ x \neq \tfrac{5}{9}\right\} \\
&= \left(\tfrac{5}{9}, \tfrac{2}{3}\right) \cup \left(\tfrac{2}{3}, \infty\right)
\end{aligned}$$

**7c** We find $(\beta \circ \beta)(x) = \beta(\beta(x)) = \beta(\sqrt{9x-5}) = \sqrt{9\sqrt{9x-5}-5}$, and the domain is given by

$$\begin{aligned}\mathrm{Dom}(\beta \circ \beta) &= \{x \mid x \in \mathrm{Dom}(\beta) \;\;\&\;\; \beta(x) \in \mathrm{Dom}(\beta)\} = \left\{x \mid x \geq \tfrac{5}{9} \;\;\&\;\; \sqrt{9x-5} \geq \tfrac{5}{9}\right\} \\ &= \left\{x \mid x \geq \tfrac{5}{9} \;\;\&\;\; x \geq \tfrac{430}{729}\right\} = \left\{x \mid x \geq \tfrac{430}{729}\right\} = \left[\tfrac{430}{729}, \infty\right),\end{aligned}$$

since $\frac{430}{729} \approx 0.59$ is slightly larger than $\frac{5}{9} \approx 0.56$.

**7d** We find $(\beta \circ \gamma)(x) = \beta(\gamma(x)) = \beta(\sqrt{36-x^2}) = \sqrt{9\sqrt{36-x^2}-5}$, and the domain is given by

$$\begin{aligned}\mathrm{Dom}(\beta \circ \gamma) &= \{x \mid x \in \mathrm{Dom}(\gamma) \;\&\; \gamma(x) \in \mathrm{Dom}(\beta)\} \\ &= \left\{x \mid -6 \leq x \leq 6 \;\;\&\;\; \sqrt{36-x^2} \geq \tfrac{5}{9}\right\} \\ &= \left\{x \mid -6 \leq x \leq 6 \;\;\&\;\; x^2 \leq \tfrac{2891}{81}\right\} \\ &= \left\{x \mid -6 \leq x \leq 6 \;\;\&\;\; -\tfrac{7\sqrt{59}}{9} \leq x \leq \tfrac{7\sqrt{59}}{9}\right\} \\ &= \left[-\tfrac{7\sqrt{59}}{9}, \tfrac{7\sqrt{59}}{9}\right]\end{aligned}$$

since $\frac{7\sqrt{59}}{9} \approx 5.97 < 6$.

**8** Let $f(x) = 2/x^{10}$ and $g(x) = 7-2x$. Then

$$(f \circ g)(x) = f(g(x)) = f(7-2x) = \frac{2}{(7-2x)^{10}} = T(x).$$

**9** Suppose that $f(a) = f(b)$. Then $2a^3 - 1 = 2b^3 - 1 \;\Rightarrow\; 2a^3 = 2b^3 \;\Rightarrow\; a^3 = b^3 \;\Rightarrow\; a = b$. Therefore $f$ is one-to-one.

**10** $g(5) = 0 = g(-9)$

**11a** Suppose that $f(x) = y$. Then $y = \dfrac{x+1}{x-3} \;\Rightarrow\; xy - 3y = x + 1 \;\Rightarrow\; xy - x = 3y + 1 \;\Rightarrow\; x = \dfrac{3y+1}{y-1}$, and since $f^{-1}(y) = x$ by definition, it follows that $f^{-1}(y) = \dfrac{3y+1}{y-1}$.

**11b** $\mathrm{Ran}(f) = \mathrm{Dom}(f^{-1}) = (-\infty, 1) \cup (1, \infty)$

**11c** $\mathrm{Ran}(f^{-1}) = \mathrm{Dom}(f) = (-\infty, 3) \cup (3, \infty)$