Math 120 Exam #2 Key (Winter 2013)

1a We have

$$\begin{aligned} 6x^2 - 11x - 7 = 0 \;&\Rightarrow\; (3x-7)(2x+1) = 0 \\ &\Rightarrow\; 3x - 7 = 0 \quad \text{or} \quad 2x + 1 = 0 \\ &\Rightarrow\; x = 7/3 \quad \text{or} \quad x = -1/2, \end{aligned}$$

so solution set is $\{7/3, -1/2\}$.

1b From $2x^2 - 4x = 3$ we get

$$\begin{aligned} x^2 - 2x = \frac{3}{2} \;&\Rightarrow\; x^2 - 2x + 1 = \frac{3}{2} + 1 \;\Rightarrow\; (x-1)^2 = \frac{5}{2} \\ &\Rightarrow\; x - 1 = \pm\sqrt{\frac{5}{2}} \;\Rightarrow\; x = 1 \pm \frac{\sqrt{10}}{2}. \end{aligned}$$

Solution set is $\left\{1 \pm \frac{\sqrt{10}}{2}\right\}$.

1c From $x^3 - 125 = 0$ we get

$$x^3 - 5^3 = 0 \;\Rightarrow\; (x-5)(x^2 + 5x + 25) = 0 \;\Rightarrow\; x - 5 = 0 \quad \text{or} \quad x^2 + 5x + 25 = 0,$$

and so by the quadratic formula we obtain

$$x = 5 \quad \text{or} \quad x = \frac{-5 \pm \sqrt{5^2 - 4(1)(25)}}{2(1)} = \frac{-5 \pm \sqrt{-75}}{2} = \frac{-5 \pm 5i\sqrt{3}}{2} = -\frac{5}{2} \pm \frac{5\sqrt{3}}{2}i.$$

Solution set is $\left\{5, -\frac{5}{2} \pm \frac{5\sqrt{3}}{2}i\right\}$.

2 Volume=(depth)(length)(width), so if ℓ is the length we have $2.3125\ell(\ell - 3.1875) = 182.742$, which is the quadratic equation

$$2.3125\ell^2 - 7.3711\ell - 182.742 = 0.$$

By the quadratic formula we get

$$\ell = \frac{7.3711 \pm \sqrt{(-7.3711)^2 - 4(2.3125)(-182.742)}}{2(2.3125)} = \frac{7.3711 \pm 41.7696}{4.6250} = 10.625, -7.438$$

Length cannot be negative, so we conclude that $\ell = 10.625$ inches. Width is therefore $10.625 - 3.1875 = 7.438$ inches. Dimensions of the box are $10.625 \times 7.438 \times 2.3125$ inches.

3 Let x be the width of the border. Since the area of the kitchen is 120 ft^2 and the area of the border must be 21 ft^2, the area of the vinyl inside the border must be 99 ft^2. The dimensions of the rectangle inside the border are $12 - 2x$ by $10 - 2x$, and so

$$(12 - 2x)(10 - 2x) = 99$$

is the equation. A little manipulation yields $4x^2 - 44x + 21 = 0$, which factors as

$$(2x-1)(2x-21) = 0,$$

and so either $x = 1/2$ or $x = 21/2$. But a border width of 21/2 ft is impossible, so we conclude that the border must be 1/2 ft wide.

4a Multiply by $x - 4$ to get $x = 4 + 4(x-4)$. From this we obtain $3x = 12$, and thus $x = 4$. But this solution is extraneous, so solution set is $\varnothing$.

4b Multiply by $x(x-2)$ to get

$$4x^2 + 3(x-2) = -6,$$

which is the quadratic equation $4x^2 + 3x = 0$. Factoring, we get $x(4x+3) = 0$, so either $x = 0$ or $x = -3/4$. Observing that 0 is an extraneous solution, we conclude that the solution set is $\{-3/4\}$.

4c Square both sides to get $2x + 3 = (x+2)^2$, so

$$2x + 3 = x^2 + 4x + 4 \quad \Rightarrow \quad x^2 + 2x + 1 = 0 \quad \Rightarrow \quad (x+1)^2 = 0 \quad \Rightarrow \quad x = -1.$$

Solution set is $\{-1\}$.

4d Square both sides to get $(3 - \sqrt{x})^2 = 2\sqrt{x} - 3$. Then

$$9 - 6\sqrt{x} + x = 2\sqrt{x} - 3 \quad \Rightarrow \quad 8\sqrt{x} = x + 12,$$

and squaring again gives

$$\begin{aligned} 64x = (x+12)^2 \quad &\Rightarrow \quad 64x = x^2 + 24x + 144 \quad \Rightarrow \quad x^2 - 40x + 144 = 0 \\ &\Rightarrow \quad (x-36)(x-4) = 0 \quad \Rightarrow \quad x = 4, 36. \end{aligned}$$

But 36 is extraneous, so solution set is $\{4\}$.

4e Factor $x^4 - 3x^2 - 4$ to obtain $(x^2-4)(x^2+1) = 0$. The equation is satisfied if $x^2 - 4 = 0$ or $x^2 + 1 = 0$. From $x^2 - 4 = 0$ we get $x = \pm 2$. From $x^2 + 1 = 0$ we get $x = \pm i$. Solution set: $\{-2, 2, -i, i\}$.

4f Either $2x + 9 = 3 - x$ or $2x + 9 = -(3-x)$. From $2x + 9 = 3 - x$ we have $x = -2$, and from $2x + 9 = -(3-x)$ we have $x = -12$. Solution set: $\{-12, -2\}$.

5a $6x - 2x - 3 \geq 3x - 5 \quad \Rightarrow \quad 4x - 3 \geq 3x - 5 \quad \Rightarrow \quad x \geq -2$, so solution set is $[-2, \infty)$.

5b $-9 < x - 1 < 6 \quad \Rightarrow \quad -8 < x < 7$, so solution set is $(-8, 7)$.

5c $6x^2 - 11x - 10 < 0 \quad \Rightarrow \quad (3x+2)(2x-5) < 0$. Case 1: $3x + 2 < 0$ & $2x - 5 > 0$, which leads to a contradiction. Case 2: $3x + 2 > 0$ & $2x - 5 < 0$, which leads to $-\frac{2}{3} < x < \frac{5}{2}$.

Solution set: $\left(-\frac{2}{3}, \frac{5}{2}\right)$.

5d $2x^3 - 3x^2 - 5x \le 0 \quad \Rightarrow \quad x(2x-5)(x+1) \le 0$.

$$\text{Case 1: } x \le 0,\ 2x - 5 \ge 0,\ x + 1 \ge 0,$$

which leads to contradiction.

$$\text{Case 2: } x \ge 0,\ 2x - 5 \le 0,\ x + 1 \ge 0,$$

which leads to $0 \le x \le \frac{5}{2}$.

$$\text{Case 3: } x \ge 0,\ 2x - 5 \ge 0,\ x + 1 \le 0,$$

again contradictory.

$$\text{Case 4: } x \le 0,\ 2x - 5 \le 0,\ x + 1 \le 0,$$

which leads to $x \le -1$.

Solution set: $(-\infty, -1] \cup \left[0, \frac{5}{2}\right]$.

5e Manipulating, we have

$$\frac{10}{2x-3} \le 5 \quad \Rightarrow \quad \frac{10}{2x-3} - \frac{5(2x-3)}{2x-3} \le 0 \quad \Rightarrow \quad \frac{25-10x}{2x-3} \le 0.$$

Case 1: $25 - 10x \le 0$ & $2x - 3 > 0$, which yields $x \ge \frac{5}{2}$ & $x > \frac{3}{2}$, and therefore $x \ge \frac{5}{2}$. Case 2: $25 - 10x \ge 0$ & $2x - 3 < 0$, which yields $x \le \frac{5}{2}$ & $x < \frac{3}{2}$, and therefore $x \ge \frac{3}{2}$.

Solution set: $\left(-\infty, \frac{3}{2}\right) \cup \left[\frac{5}{2}, \infty\right)$.

5f From $|8x - 3| > 5$ we obtain

$$8x - 3 > 5 \quad \text{or} \quad 8x - 3 < -5 \quad \Rightarrow \quad 8x > 8 \quad \text{or} \quad 8x < -2 \quad \Rightarrow \quad x > 1 \quad \text{or} \quad x < -1/4.$$

So the solution set is $(-\infty, -1/4) \cup (1, \infty)$.

5g It is impossible for the absolute value of a real number to be less than 0, so there is no solution to the inequality $|10x - 3| \le -2$. Solution set is $\varnothing$.

6 Distance $= \sqrt{(6+3)^2 + (-2-8)^2} = \sqrt{9^2 + 10^2} = \sqrt{181}$.

7 Three ordered pairs in the solution set are $(0, \sqrt{2})$, $(2, 2)$, $(-1, 1)$. There are many others.

8 From $(x^2 - 12) + (y^2 + 10y) = -25$ we obtain

$$(x^2 - 12x + 36) + (y^2 + 10y + 25) = -25 + 36 + 25 \quad \Rightarrow \quad (x-6)^2 + (y+5)^2 = 36,$$

which is a circle with center at $(6, -5)$ and radius 6.

9 Employ a graphical approach as in the example in the textbook: at coordinates $(7, 4)$ graph a circle of radius 5, at $(-9, -4)$ graph a circle of radius 13, and at $(-3, 9)$ graph a circle of radius 10. Looking at the graph below, only the one point $(3, 1)$ lies on all three circles, and so the epicenter of the earthquake must be at $(3, 1)$.