MATH 120 EXAM #4 KEY (SUMMER II - 2010)

1a. We can get a common base of 2 (or 4): $(2^4)^{2x-1} = (2^6)^{3x} \Rightarrow 2^{8x-4} = 2^{18x} \Rightarrow 8x - 4 = 18x \Rightarrow x = -\frac{2}{5}$.

1b. Cube both sides to get $r^2 = 4^3$, so $r^2 = 64$ and we obtain $r = \pm 8$.

1c. Convert to an exponential equation to get $8^y = \sqrt[4]{8} \Rightarrow 8^y = 8^{1/4} \Rightarrow y = \frac{1}{4}$.

1d. Convert to an exponential equation to get $x^{-1} = 3$, or $x = \frac{1}{3}$.

2a. $\ln 6^{x+3} = \ln 4^x \Rightarrow (x+3)\ln 6 = x \ln 4 \Rightarrow x \ln 6 + 3 \ln 6 = x \ln 4 \Rightarrow x(\ln 4 - \ln 6) = 3 \ln 6 \Rightarrow x = \dfrac{3 \ln 6}{\ln 4 - \ln 6} \approx -13.2571$.

2b. $(1.05)^{x/4} = \dfrac{2}{5} \Rightarrow \dfrac{x}{4} \ln 1.05 = \ln 0.4 \Rightarrow x = \dfrac{4 \ln 0.4}{\ln 1.05} \approx -75.1209$.

2c. Since the logarithmic function is one-to-one, we immediately obtain $3x + 8 = 18$, and thus $x = \frac{10}{3}$.

2d. Combining logarithms gives $\log_2(x^2 + 2x) = 3$, which implies that $2^3 = x^2 + 2x$. Solving this equation yields $x = -4, 2$. But -4 is an extraneous solution (it results in a logarithm of a negative number in the original equation), so the only valid solution is $x = 2$.

2e. We get $2 \log x - (\log x)^2 = 0 \Rightarrow (\log x)(2 - \log x) = 0$. So either $\log x = 0$ or $\log x = 2$, from which we obtain $x = 1$ or $x = 100$. Both solutions are valid, so the solution set is $\{1,\ 100\}$.

3. Using our handy-dandy patent-pending little formula $A = P(1 + r/m)^{mt}$ we get $2000 = 1200(1 + r/4)^{4\cdot 5}$, whence $\frac{5}{3} = (1 + r/4)^{20} \Rightarrow 1 + r/4 = \sqrt[20]{5/3} \Rightarrow r = 4(\sqrt[20]{5/3} - 1) \approx 0.10348$, or 10.3%.

4. So if we start with P clams we want to know how long it'll take to have $3P$ clams. Set $A = 3P$ and $r = 0.05$ in the formula $A = Pe^{rt}$ and get $3P = Pe^{0.05t}$. Then $e^{0.05t} = 3 \Rightarrow 0.05t = \ln 3 \Rightarrow t = \frac{\ln 3}{0.05} \approx 21.97 \approx 22$ years.

5a. Three weeks means 21 days, so: $A(21) = 500e^{-0.0012(21)} \approx 487.6$ g.

5b. We're starting with 500 grams, so we need to find the time t for which $A(t) = 250$ g. Solve $250 = 500e^{-0.0012t}$ to get $e^{-0.0012t} = 0.5 \Rightarrow -0.0012t = \ln 0.5 \Rightarrow t = \frac{\ln 0.5}{-0.0012} \approx 577.6$ days.

6. This is the problem your mother warned you about. Our initial amount of kaboomium is $A_0 = 200$ g, meaning our basic model is $A(t) = 200e^{-kt}$ and it remains to find k. We're given that $A(4) = 192$ g, which is to say that $192 = 200e^{-k\cdot 4}$. Solve this equation: $e^{-4k} = 0.96 \Rightarrow -4k = \ln 0.96 \Rightarrow k = \frac{\ln 0.96}{-4} \approx 0.010205$. Thus the model here is $A(t) = 200e^{-0.010205t}$, and it's now possible to find the time t for which just 12 g of kaboomium remains: $12 = 200e^{-0.010205t} \Rightarrow e^{-0.010205t} = 0.06 \Rightarrow -0.010205t = \ln 0.06 \Rightarrow t = \frac{\ln 0.06}{-0.010205} \approx 275.7$ hours. We assume Cobra Commander knows his business. And knowing is half the battle!

7a. From the first equation we obtain $x = \frac{1}{2}(3y - 7)$, which can then be substituted into the second equation to give $5 \cdot \frac{1}{2}(3y - 7) + 4y = 17$. Solving this yields $y = 3$. Now put this result into $x = \frac{1}{2}(3y - 7)$ to get $x = 1$. The solution is $(3, 1)$.

7b. The 3rd equation gives $y = 2x - 4z + 14$. Putting this into the 1st and 2nd equations yields $4z - (2x - 4z + 14) + 3z = -2$ and $3x + 5(2x - 4z + 14) - z = 15$, respectively. Simplifying gives us the system

$$\begin{cases} 2x + 7z = 12 \\ 13x - 21z = -55 \end{cases}$$

This system solves to give $x = -1$ and $z = 4$. Putting these values into $y = 2x - 4z + 14$ gives $y = -4$. Hence the solution is $(-1, -4, 4)$.

Extra Credit