MATH 120 EXAM #3 KEY (SUMMER II - 2010)

1. Notice that $(1,-1)$ and $(1,1)$ belong to the relation, which are two distinct ordered pairs with the same 1st component value—something which a function cannot have. Domain of relation = possible x values = $[0,\infty)$. Range of relation = possible y values = $(-\infty,\infty)$.

2. $f(-4)=(-4)^2-3(-4)+1=16+12+1=29$, and $f(c+2)=(c+2)^2-3(c+2)+1=c^2+c-1$.

3a. Dom $f=[-3,5]$ and Ran $f=[2,27]$.

3b. Dom $f=\{x \mid f(x)\in\mathbb{R}\}=\{x \mid \sqrt{7-3x}$ is real $\}=\{x \mid 7-3x\geqslant 0\}=\{x \mid x\leqslant 7/3\}=(-\infty,7/3]$ and Ran $f=[0,\infty)$.

4a. $(\varphi+\psi)(x)=\dfrac{8}{x-4}+\dfrac{x+1}{x-2}$ and $(\varphi/\psi)(x)=\dfrac{\varphi(x)}{\psi(x)}=\dfrac{8/(x-4)}{(x+1)/(x-2)}=\dfrac{8(x-2)}{(x-4)(x+1)}$.

4b. Dom $\varphi=(-\infty,4)\cup(4,\infty)$ and Dom $\psi=(-\infty,2)\cup(2,\infty)$.

4c. Dom $\varphi/\psi=$ Dom $\varphi\cap$ Dom $\psi-\{x \mid \psi(x)=0\}=\{x \mid x\neq 2,4\}-\{-1\}=\{x \mid x\neq -1,2,4\}$.

5a. $(\alpha\circ\beta)(x)=\alpha(\sqrt{x+8})=\sqrt{9-(\sqrt{x+8})^2}=\sqrt{1-x}$ and $(\alpha\circ\alpha)(x)=\alpha(\sqrt{9-x^2})=\sqrt{9-(\sqrt{9-x^2})^2}=\sqrt{9-(9-x^2)}=\sqrt{x^2}=|x|$.

5b. Dom $\alpha=\{x \mid 9-x^2\geqslant 0\}=\{x \mid x^2\leqslant 9\}=[-3,3]$ and Dom $\beta=\{x \mid x+8\geqslant 0\}=\{x \mid x\geqslant -8\}=(-8,\infty)$.

5c. By definition, Dom $(\alpha\circ\beta)=\{x \mid x\in$ Dom β and $\beta(x)\in$ Dom $\alpha\}$, where $x\in$ Dom β implies that $x\geqslant -8$, and $\beta(x)\in$ Dom $\alpha \;\Rightarrow\; \sqrt{x+8}\in[-3,3] \;\Rightarrow\; -3\leqslant\sqrt{x+8}\leqslant 3 \;\Rightarrow\; 0\leqslant x+8\leqslant 9 \;\Rightarrow\; -8\leqslant x\leqslant 1$. Therefore we have Dom $(\alpha\circ\beta)=\{x \mid x\geqslant -8$ and $-8\leqslant x\leqslant 1\}$, or more simply: Dom $(\alpha\circ\beta)=[-8,1]$.

6. One way that works: let $h(x)=x-7$, $g(x)=\dfrac{12}{x^5}$ and $f(x)=x^8$. Then $(f\circ g\circ h)(x)=f(g(h(x)))=f(g(x-7))=f\left(\dfrac{12}{(x-7)^5}\right)=\left[\dfrac{12}{(x-7)^5}\right]^8=W(x)$.

7. Factoring yields $z(x)=(x+7)(x-3)$, so it can be seen that $z(3)=0=z(-7)$, which immediately shows that z is not one-to-one.

8a. Set $f(x)=y$ to get $y=x^3-7$. Solving for x, we have $x^3=y+7$ and finally $x=\sqrt[3]{y+7}$. Since $x=f^{-1}(y)$ we find that $f^{-1}(y)=\sqrt[3]{y+7}$, or equivalently $f^{-1}(x)=\sqrt[3]{x+7}$. Domain of f and range of f^{-1}: $(-\infty,\infty)$. Range of f and domain of f^{-1}: $(-\infty,\infty)$.

8b. Set $f(x)=y$ to get $y=\dfrac{x-4}{x+5}$. Solving for x, we have $y(x+5)=x-4 \;\Rightarrow\; xy-x=-4-5y \;\Rightarrow\; x(y-1)=-5y-4 \;\Rightarrow\; x=-\dfrac{5y+4}{y-1}$. Since $x=f^{-1}(y)$ we find that $f^{-1}(y)=-\dfrac{5y+4}{y-1}$, or equivalently $f^{-1}(x)=-\dfrac{5x+4}{x-1}$. Domain of f and range of f^{-1}: $(-\infty,-5)\cup(-5,\infty)$. Range of f and domain of f^{-1}: $(-\infty,1)\cup(1,\infty)$.

9. $\begin{array}{r|rrrrr|r} -2 & 1 & 3 & 2 & 2 & 3 & 1 \\ & & -2 & -2 & 0 & -4 & 2 \\ \hline & 1 & 1 & 0 & 2 & -1 & 3 \end{array}$ $\longrightarrow$ so the result is: $x^4 + x^3 + 2x - 1 + \dfrac{3}{x+2}$.

10a. $\dfrac{\pm 1, \pm 3}{1, 2, 4, 8} = \pm 1,\ \pm\frac{1}{2},\ \pm\frac{1}{4},\ \pm\frac{1}{8},\ \pm 3, \pm\frac{3}{2},\ \pm\frac{3}{4},\ \pm\frac{3}{8}.$

10b. $\begin{array}{r|rrrr|r} 3 & 8 & -14 & -29 & -4 & 3 \\ & & 24 & 30 & 3 & -3 \\ \hline & 8 & 10 & 1 & -1 & 0 \end{array}$ and $\begin{array}{r|rrr|r} -1 & 8 & 10 & 1 & -1 \\ & & -8 & -2 & 1 \\ \hline & 8 & 2 & -1 & 0 \end{array}$ show that 3 and -1 are zeros for the function f, and we obtain the factorization $f(x) = (x-3)(x+1)(8x^2 + 2x - 1)$ from the bottom row of numbers in the second synthetic division. Now $f(x) = 0$ implies (by the Zero Factor Principle from way back) that $x - 3 = 0$ or $x + 1 = 0$ or $8x^2 + 2x - 1 = 0$. The last equation yields two new zeros: $\frac{1}{4}$ and $-\frac{1}{2}$. So the zeros of f are: 3, -1, $\frac{1}{4}$, $-\frac{1}{2}$.

10c. From part (b) it's easy to see that $f(x) = (x-3)(x+1)(4x-1)(2x+1)$.

11. We must have $f(x) = c(x+2)(x-1)(x-4)$ by the Factor Theorem, where c will be an appropriate constant that will give $f(2) = 16$. But $f(2) = 16$ gives us $c(2+2)(2-1)(2-4) = 16$, or $c = -2$. Hence $f(x) = -2(x+2)(x-1)(x-4)$.

12. To have real coefficients the Conjugate Zeros Theorem implies that $2 - i$ must also be a zero. Then, by the Factor Theorem, we obtain $f(x) = (x-3)[x-(2+i)][x-(2-i)] = (x-3)(x^2 - 4x + 5)$, or $f(x) = x^3 - 7x^2 + 17x - 15$.