Math 120 Exam #3 Key (Summer 2015)

1 Solve $-6x + 2y = 3$ for y to get $y = 3x + \frac{3}{2}$, which shows the slope for the line given by $-6x + 2y = 3$ to be 3. Since the line L through $(5, -3)$ is parallel to this line, its slope must also be 3. So the equation for L is $y - (-3) = 3(x - 5)$, or

$$y = 3x - 18.$$

2 Solve $4x - 3y = 6$ for y to get $y = \frac{4}{3}x - 2$, which shows the slope for the line given by $4x - 3y = 6$ to be $\frac{4}{3}$. Since the line L through $(1, 9)$ is perpendicular to this line, its slope must be $-\frac{3}{4}$. So the equation for L is $y - 9 = -\frac{3}{4}(x - 1)$, or

$$y = -\tfrac{3}{4}x + \tfrac{39}{4}.$$

3 The line must be vertical and pass through $(30, 16)$ where x is 30. Equation is $x = 30$.

4a Domain is $[0, \infty)$ and range is $[3, \infty)$

4b Domain is $(-\infty, \infty)$ and range is $(-\infty, 7]$

5a $\mathrm{Dom}(\varphi) = (-\infty, 4) \cup (4, \infty)$

5b $\mathrm{Dom}(\omega) = \{x : x + 4 \geq 0\} = \{x : x \geq -4\} = [-4, \infty)$

5c $\mathrm{Dom}(\psi) = \{x : 81 - x^2 \geq 0\} = \{x : x^2 \leq 81\} = \{x : -9 \leq x \leq 9\} = [-9, 9]$

6a By definition

$$(\varphi + \psi)(x) = \varphi(x) + \psi(x) = \frac{x+1}{4-x} + \sqrt{81 - x^2},$$

and

$$\mathrm{Dom}(\varphi + \psi) = \mathrm{Dom}(\varphi) \cap \mathrm{Dom}(\psi) = \big[(-\infty, 4) \cup (4, \infty)\big] \cap [-9, 9] = [-9, 4) \cup (4, 9].$$

6b By definition

$$(\varphi/\omega)(x) = \varphi(x)/\omega(x) = \frac{x+1}{4-x} \cdot \frac{1}{\sqrt{x+4}} = \frac{x+1}{(4-x)\sqrt{x+4}},$$

and

$$\begin{aligned}\mathrm{Dom}(\varphi/\omega) &= \{x : x \in \mathrm{Dom}(\varphi) \cap \mathrm{Dom}(\omega) \text{ and } \omega(x) \neq 0\} \\ &= \{x : x \in (-\infty, 4) \cup (4, \infty) \text{ and } x \in [-4, \infty) \text{ and } x \neq -4\} \\ &= (-4, 4) \cup (4, \infty).\end{aligned}$$

6c By definition

$$(\omega \circ \omega)(x) = \omega(\omega(x)) = \omega\left(\sqrt{x+4}\right) = \sqrt{\sqrt{x+4}+4},$$

and

$$\begin{aligned}
\mathrm{Dom}(\omega \circ \omega) &= \{x : x \in \mathrm{Dom}(\omega) \text{ and } \omega(x) \in \mathrm{Dom}(\omega)\} \\
&= \left\{x : x \geq -4 \text{ and } \sqrt{x+4} \geq -4\right\} \\
&= \{x : x \geq -4\} = [-4, \infty).
\end{aligned}$$

6d By definition

$$(\omega \circ \psi)(x) = \omega(\psi(x)) = \omega\big(\sqrt{81-x^2}\,\big) = \sqrt{\sqrt{81-x^2}+4},$$

and

$$\begin{aligned}
\mathrm{Dom}(\omega \circ \psi) &= \{x : x \in \mathrm{Dom}(\psi) \text{ and } \psi(x) \in \mathrm{Dom}(\omega)\} \\
&= \big\{x : -9 \leq x \leq 9 \text{ and } \sqrt{81-x^2} \geq -4\big\} \\
&= \big\{x : -9 \leq x \leq 9 \text{ and } x^2 \leq 81\big\} \\
&= \{x : -9 \leq x \leq 9\} = [-9, 9].
\end{aligned}$$

7 Let $f(x) = \sqrt[5]{x}$ and $g(x) = 2x - 9$. Then

$$(f \circ g)(x) = f(g(x)) = f(2x-9) = \sqrt[5]{2x-9} = H(x).$$

8 Suppose that $f(a) = f(b)$. Then

$$2a^3 - 1 = 2b^3 - 1 \quad \Rightarrow \quad 2a^3 = 2b^3 \quad \Rightarrow \quad a^3 = b^3 \quad \Rightarrow \quad a = b.$$

Therefore f is one-to-one.

9 Since $g(0) = 1^4 + 1^2 = 2 = (-1)^4 + (-1)^2 = g(-1)$, g is not one-to-one.

10a Suppose that $f(x) = y$. Then

$$y = \frac{4}{x} \quad \Rightarrow \quad x = \frac{4}{y}$$

and since $f^{-1}(y) = x$ by definition, it follows that

$$f^{-1}(y) = \frac{4}{y}.$$

10b $\mathrm{Ran}(f) = \mathrm{Dom}(f^{-1}) = (-\infty, 0) \cup (0, \infty)$

10c $\mathrm{Ran}(f^{-1}) = \mathrm{Dom}(f) = (-\infty, 0) \cup (0, \infty)$