Math 120 Exam #2 Key (Summer 2015)

1a We have

$$5x^2 - 3x - 2 = 0 \quad\Rightarrow\quad (5x+2)(x-1) = 0 \quad\Rightarrow\quad 5x+2=0 \quad\text{or}\quad x-1=0,$$

so the solution set is $\{1, -2/5\}$

1b We have

$$x^2 - 3x = 6 \quad\Rightarrow\quad x^2 - 3x + \tfrac{9}{4} = 6 + \tfrac{9}{4} \quad\Rightarrow\quad \left(x - \tfrac{3}{2}\right)^2 = \tfrac{33}{4} \quad\Rightarrow\quad x - \tfrac{3}{2} = \pm\tfrac{\sqrt{33}}{2} \quad\Rightarrow\quad x = \tfrac{3}{2} \pm \tfrac{\sqrt{33}}{2},$$

and so the solution set is $\left\{\frac{3}{2} - \frac{\sqrt{33}}{2}, \frac{3}{2} + \frac{\sqrt{33}}{2}\right\}$.

2 If w is the width of the metal sheet, then the length of the sheet is $w + 10$. However, the *box* has width $w - 4$ and length $(w + 10) - 4 = w + 6$, and the height must be 2. The volume V of the box is computed as $V = 2(w - 4)(w + 6)$, but we're also given that $V = 832$. This gives us an equation: $2(w - 4)(w + 6) = 832$. Hence $w^2 + 2w - 440 = 0$, which leads to $(w + 22)(w - 20) = 0$ and so $w = -22, 20$. Clearly the width of the original sheet can't be -22 cm, which leaves it to be 20 cm. Dimensions of sheet are 20 cm $\times$ 30 cm.

3 Let w be the width, so the length is $w + 6$. By the Pythagorean Theorem the length of the diagonal is $\sqrt{w^2 + (w+6)^2}$, which is given to be 174, so that

$$w^2 + (w+6)^2 = 174^2 \quad\Rightarrow\quad w^2 + 6w - 15{,}120 = 0 \quad\Rightarrow\quad (w - 120)(w - 126) = 0.$$

Thus the width is 120, and then the length is 126. Dimensions are 126 m $\times$ 120 m.

4

	Rate of Work	Time Worked	Fraction of Job Done
Taps	$\dfrac{1}{6}$	t	$\dfrac{t}{6}$
Drain	$-\dfrac{1}{10}$	t	$-\dfrac{t}{10}$

Let t be the time it would take to complete the job of filling the sink. We get

$$\frac{t}{6} - \frac{t}{10} = 1 \quad\Rightarrow\quad 5t - 3t = 30 \quad\Rightarrow\quad 2t = 30 \quad\Rightarrow\quad t = 15 \text{ minutes.}$$

Get your head screwed on straight, Harry!

5a We have

$$x(x+2)\left(\frac{x}{x+2} + \frac{1}{x} + 3\right) = x(x+2)\cdot\frac{2}{x(x+2)} \quad\Rightarrow\quad x^2 + (x+2) + 3x(x+2) = 2 \quad\Rightarrow\quad 4x^2 + 7x = 0,$$

giving $x(4x + 7) = 0$, and finally $x = 0, -\frac{7}{4}$. However, 0 is an extraneous solution, so the solution set is $\left\{-\frac{7}{4}\right\}$.

5b Solving,

$$\sqrt{2x} = x - 4 \quad\Rightarrow\quad 2x = (x-4)^2 \quad\Rightarrow\quad x^2 - 10x + 16 = 0 \quad\Rightarrow\quad (x-8)(x-2) = 0,$$

and so $x = 2$ or $x = 8$. But 2 is extraneous (it gives us $2 = -2$ in the original equation), so solution set is $\{8\}$.

5c We have

$$\sqrt{x} = \sqrt{x+3} - 1 \;\Rightarrow\; x = (\sqrt{x+3} - 1)^2 \;\Rightarrow\; x = (x+3) - 2\sqrt{x+3} + 1$$
$$\Rightarrow\; 2\sqrt{x+3} = 4 \;\Rightarrow\; 4(x+3) = 16 \;\Rightarrow\; x = 1.$$

Solution set is $\{1\}$.

5d We have

$$3(r^2)^2 + 10r^2 - 25 = 0 \;\Rightarrow\; (r^2+5)(3r^2-5) = 0 \;\Rightarrow\; r^2 = -5 \quad \text{or} \quad r^2 = \frac{5}{3},$$

so $r = \pm i\sqrt{5}$ or $r = \pm\sqrt{5/3}$. Solution set is $\left\{\pm i\sqrt{5}, \pm\sqrt{5/3}\right\}$.

5e Solving

$$|18 - 4y| = 15 \;\Rightarrow\; 18 - 4y = \pm 15 \;\Rightarrow\; y = \frac{-18 \pm 15}{-4},$$

so the solution set is $\left\{\frac{3}{4}, \frac{33}{4}\right\}$.

6a Solving,

$$6x - (2x + 13) \geq 3x - 5 \;\Rightarrow\; 4x - 13 \geq 3x - 5 \;\Rightarrow\; x \geq 8,$$

so solution set is $[8, \infty)$.

6b $-18 < x - 4 < 12 \;\Rightarrow\; -14 < x < 16$, so solution set is $(-14, 16)$.

6c We get $x^2 + 5x + 7 < 0$, which is not directly factorable. Consider a completing the square procedure:

$$x^2 + 5x + 7 = \left(x^2 + 5x + \tfrac{25}{4}\right) + 7 - \tfrac{25}{4} = \left(x + \tfrac{5}{2}\right)^2 + \tfrac{3}{4}.$$

Thus $x^2 + 5x + 7 < 0$ is equivalent to

$$\left(x + \tfrac{5}{2}\right)^2 + \tfrac{3}{4} < 0,$$

which has no solution. Solution set is therefore $\varnothing$.

6d There are four cases:

$$\begin{array}{lllllll}
\text{Case 1:} & t+5 \geq 0 & \text{and} & 3t-4 \geq 0 & \text{and} & t+2 \geq 0. \\
\text{Case 2:} & t+5 \geq 0 & \text{and} & 3t-4 \leq 0 & \text{and} & t+2 \leq 0. \\
\text{Case 3:} & t+5 \leq 0 & \text{and} & 3t-4 \geq 0 & \text{and} & t+2 \leq 0. \\
\text{Case 4:} & t+5 \leq 0 & \text{and} & 3t-4 \leq 0 & \text{and} & t+2 \geq 0.
\end{array}$$

Solving we have

$$\text{Case 1:} \quad t \geq -5 \quad \text{and} \quad t \geq 4/3 \quad \text{and} \quad t \geq -2.$$

$$\begin{array}{lllllll} \text{Case 2:} & t \geq -5 & \text{and} & t \leq 4/3 & \text{and} & t \leq -2. \\ \text{Case 3:} & t \leq -5 & \text{and} & t \geq 4/3 & \text{and} & t \leq -2. \\ \text{Case 4:} & t \leq -5 & \text{and} & t \leq 4/3 & \text{and} & t \geq -2. \end{array}$$

To satisfy the conditions of Case 1 we need $t \geq 4/3$, and to satisfy the conditions of Case 2 we need $-5 \leq t \leq -2$. Case 3 is impossible to realize, since we can never have $t \leq -5$ *and* $t \geq 4/3$ simultaneously. Also Case 4 is impossible since we can never have $t \leq -5$ *and* $t \geq -2$ simultaneously. So we must have either $t \geq 4/3$ or $-5 \leq t \leq -2$, giving $[-5, -2] \cup [4/3, \infty)$ for the solution set.

6e First some manipulation is in order:

$$\frac{4}{2-x} \geq \frac{3}{1-x} \quad \Rightarrow \quad \frac{4}{2-x} - \frac{3}{1-x} \geq 0 \quad \Rightarrow \quad -\frac{x+2}{(2-x)(1-x)} \geq 0,$$

or equivalently, multiplying both sides by -1 and noting that $(2-x)(1-x) = (x-2)(x-1)$,

$$\frac{x+2}{(x-2)(x-1)} \leq 0.$$

Noting that we cannot have $x - 2 = 0$ or $x - 1 = 0$, there are four cases:

$$\begin{array}{lllllll} \text{Case 1:} & x+2 \leq 0 & \text{and} & x-2 < 0 & \text{and} & x-1 < 0. \\ \text{Case 2:} & x+2 \leq 0 & \text{and} & x-2 > 0 & \text{and} & x-1 > 0. \\ \text{Case 3:} & x+2 \geq 0 & \text{and} & x-2 < 0 & \text{and} & x-1 > 0. \\ \text{Case 4:} & x+2 \geq 0 & \text{and} & x-2 > 0 & \text{and} & x-1 < 0. \end{array}$$

Solving, we have

$$\begin{array}{lllllll} \text{Case 1:} & x \leq -2 & \text{and} & x < 2 & \text{and} & x < 1. \\ \text{Case 2:} & x \leq -2 & \text{and} & x > 2 & \text{and} & x > 1. \\ \text{Case 3:} & x \geq -2 & \text{and} & x < 2 & \text{and} & x > 1. \\ \text{Case 4:} & x \geq -2 & \text{and} & x > 2 & \text{and} & x < 1. \end{array}$$

Case 1 gives $x \leq -2$, and Case 3 gives $1 < x < 2$. The other cases are impossible. Solution set: $(-\infty, -2] \cup (1, 2)$.

6f Solving,

$$|8x - 3| < 4 \quad \Rightarrow \quad -4 < 8x - 3 < 4 \quad \Rightarrow \quad -1 < 8x < 7 \quad \Rightarrow \quad -\tfrac{1}{8} < x < \tfrac{7}{8}.$$

Solution set: $\left(-\frac{1}{8}, \frac{7}{8}\right)$.

7 By the distance formula: $\sqrt{(-6-8)^2 + (-5-12)^2} = \sqrt{14^2 + 17^2} = \sqrt{485}$.

8 Using the completing the square technique gives

$$(x^2 + 8x + 16) + (y^2 - 6y + 9) = -16 + 16 + 9 \quad \Rightarrow \quad (x+4)^2 + (y-3)^2 = 3^2,$$

which is a circle with center at $(-4, 3)$ and radius 3.