

MATH 120 EXAM #1 KEY (SUMMER 2015)

1 $x^2 + 7y - z^4 = (-4)^2 + 7(4) - (-2)^4 = 16 + 28 - 16 = 28$

2a $(u^3 - 2u^2 + 5) - 2(-7u^3 + 11u^2) = u^3 - 2u^2 + 5 + 14u^3 - 22u^2 = 15u^3 - 24u^2 + 5$

2b $(3v + 2)(4v^2 - 7v + 6) = 12v^3 - 13v^2 + 4v + 12$

2c $(a - 8b)^2 = a^2 - 16ab + 64b^2$

3 Answer is: $4x^2 + 2x + \frac{-7x + 1}{2x^2 - x + 2}$.

$$\begin{array}{r} 2x^2 - x + 2 \overline{) \begin{array}{r} 8x^4 + 6x^2 - 3x + 1 \\ - 8x^4 + 4x^3 - 8x^2 \\ \hline 4x^3 - 2x^2 - 3x \\ - 4x^3 + 2x^2 - 4x \\ \hline - 7x \end{array}} \end{array}$$

4a $10ab - 6b + 35a - 21 = 2b(5a - 3) + 7(5a - 3) = (5a - 3)(2b + 7)$

4b $9z^2 + 4z - 2$ is prime

4c $32a^2 + 48ab + 18b^2 = 2(16a^2 + 24ab + 9b^2) = 2(4a + 3b)^2$

4d $36k^2 - 81\ell^4 = 9(4k^2 - 9\ell^4) = 9(2k - 3\ell^2)(2k + 3\ell^2)$

4e $1000x^3 + 343y^3 = (10x)^3 + (7y)^3 = (10x + 7y)(100x^2 - 70xy + 49y^2)$

5a $\frac{q^3 + q^2}{7} \cdot \frac{49}{q^4 + q^3} = \frac{q^2(q + 1)}{7} \cdot \frac{7^2}{q^3(q + 1)} = \frac{1}{1} \cdot \frac{7}{q} = \frac{7}{q}$

5b $\frac{x^2 + x - 2}{x^2 + 3x - 4} \div \frac{x^2 + 3x + 2}{x^2 + 4x + 3} = \frac{(x + 2)(x - 1)}{(x + 4)(x - 1)} \cdot \frac{(x + 3)(x + 1)}{(x + 2)(x + 1)} = \frac{x + 3}{x + 4}$

$$\mathbf{6} \quad \frac{5}{12x^2y} - \frac{7}{6xy^3} = \frac{5}{12x^2y} \cdot \frac{y^2}{y^2} - \frac{7}{6xy^3} \cdot \frac{2x}{2x} = \frac{5y^2}{12x^2y^3} - \frac{14x}{12x^2y^3} = \frac{5y^2 - 14x}{12x^2y^3}$$

7 We have

$$\frac{1 - \frac{2}{3x}}{9 - \frac{4}{x^2}} = \frac{1 - \frac{2}{3x}}{9 - \frac{4}{x^2}} \cdot \frac{3x^2}{3x^2} = \frac{3x^2 - 2x}{27x^2 - 12} = \frac{x(3x - 2)}{3(3x - 2)(3x + 2)} = \frac{x}{9x + 6}$$

$$\mathbf{8} \quad \frac{(r^{-1/5}s^{2/3})^{15}}{r^{-2}} = \frac{r^{-3}s^{10}}{r^{-2}} = \frac{s^{10}}{r}$$

$$\mathbf{9} \quad w^{-7} + 6w^{-9} = w^{-9}(w^2 + 6).$$

$$\mathbf{10a} \quad \sqrt{25j^4k^2} = 5|j^2k| = 5j^2k, \text{ since } j, k > 0.$$

10b If $x \geq 0$, since $|x| = x$,

$$\sqrt{8x^2z^8} = 2|x|z^4\sqrt{2} = 2xz^4\sqrt{2}.$$

If $x < 0$, since $|x| = -x$,

$$\sqrt{8x^2z^8} = 2|x|z^4\sqrt{2} = -2xz^4\sqrt{2}.$$

$$\mathbf{10c} \quad \sqrt[3]{\frac{9}{16r^4}} = \frac{\sqrt[3]{9}}{\sqrt[3]{16r^4}} = \frac{\sqrt[3]{9}}{2r\sqrt[3]{2r}} = \frac{\sqrt[3]{9}}{2r\sqrt[3]{2r}} \cdot \frac{\sqrt[3]{4r^2}}{\sqrt[3]{4r^2}} = \frac{\sqrt[3]{36r^2}}{4r^2}$$

$$\mathbf{10d} \quad \sqrt[5]{\sqrt[6]{60}} = \sqrt[30]{60}$$

$$\mathbf{10e} \quad \sqrt[3]{32} - 5\sqrt[3]{4} - 8\sqrt[3]{108} = 2\sqrt[3]{4} - 5\sqrt[3]{4} - 8 \cdot 3\sqrt[3]{4} = -27\sqrt[3]{4}$$

$$\mathbf{11} \quad 4[2x - (3 - x) + 5] = -6x - 28 \Rightarrow 4(3x + 2) = -6x - 28 \Rightarrow 18x = -36 \Rightarrow x = -2$$

12 We have

$$ax + b = 3x - 3a \Rightarrow ax + 3a = 3x - b \Rightarrow a(x + 3) = 3x - b \Rightarrow a = \frac{3x - b}{x + 3}.$$

13 Let x be the quantity of 60% acid solution to add. We equate the total amount of pure acid present in the two solutions before mixing with the amount of pure acid present in the final mixture:

$$0.60x + 0.45(400) = 0.55(x + 400).$$

Solving yields $x = 800$. That is, 800 mL of 60% solution must be added.

14 Let x be the amount of money invested at 5.5%, so that $12,000 - x$ is the amount invested at 4%. Add the interest from each investment to get the total interest of \$560:

$$0.055x + 0.04(12,000 - x) = 560.$$

Solving yields $x = 5,333.33$. That is, \$5,333.33 is invested at 5.5% and \$6,666.67 at 4%.

15a $(3 - 6i) - (-8 - 13i) = 11 + 7i$

15b $(4 - 3i)(2 + 9i) = 8 + 36i - 6i - 27i^2 = 35 + 30i$

15c Noting that $-i^2 = -1$, we have

$$\frac{3 - 2i}{1 + i} \cdot \frac{1 - i}{1 - i} = \frac{3 - 3i - 2i + 2i^2}{1 + i - i - i^2} = \frac{1 - 5i}{2} = \frac{1}{2} - \frac{5}{2}i.$$

15d Noting that $i^3 = i^2 \cdot i = -1 \cdot i = -i$, we have

$$i^{355} = i^{4(88)+3} = i^{4(88)} \cdot i^3 = (i^4)^{88} \cdot i^3 = 1^{88} \cdot (-i) = -i.$$