Math 120 Exam #4 Key (Spring 2024)

1a $D_f = \{x \mid 19 - 2x > 0\} = (-\infty, \frac{19}{2})$.

1b $D_f = \left\{ x \;\middle|\; \dfrac{x+5}{x^2+10} > 0 \right\} = \{x \mid x + 5 > 0\} = (-5, \infty)$.

2 $\log \dfrac{x(x^2-1)}{7(x+1)} = \log \dfrac{x(x-1)}{7}$.

3a Get $e^{x+1} = e^{-1}$, which implies $x + 1 = -1$, and so $x = -2$.

3b Letting $u = e^{2x}$ is an option, but not necessary. Factor: $(e^{2x} - 6)(e^{2x} + 3) = 0$. So either $e^{2x} = 6$ or $e^{2x} = -3$. There is no solution to $e^{2x} = -3$. From $e^{2x} = 6$ we get $2x = \ln 6$, or $x = \frac{\ln 6}{2}$.

3c Write $\ln \sqrt{x+4} = 1$, which is equivalent to $\sqrt{x+4} = e$, and hence $x = e^2 - 4$.

3d Consolidate to get $\log_9(x-5)(x+3) = 1$, which is equivalent to $9^1 = (x-5)(x+3)$, and so $x^2 - 2x - 24 = 0$. Solving the quadratic equation yields $x = -4, 6$; however, -4 is an extraneous solution for the original logarithmic equation. Solution set: $\{6\}$.

4 For $A(t) = 50e^{-kt}$ we have $\frac{1}{2} \cdot 50 = A(25) = 50e^{-25k}$, so $e^{-25k} = \frac{1}{2}$, and hence $k = 0.02773$. The completed model is now $A(t) = 50e^{-0.02773t}$, and we find t such that $A(t) = 32$. This implies

$$50e^{-0.02773t} = 32,$$

or $e^{-0.02773t} = 0.64$. Solving, we get $t \approx 16.1$ years.

5 The model will have the form $A(t) = A_0e^{-kt}$. Given is that $A(5730) = \frac{1}{2}A_0$. Thus $A_0e^{-5730k} = \frac{1}{2}A_0$, which becomes $e^{-5730k} = \frac{1}{2}$, and hence $k = \frac{\ln 2}{5730} \approx 1.210 \times 10^{-4}$. The model is now $A(t) = A_0e^{-0.0001210t}$.

We now find time t for which $A(t) = 0.71A_0$, or $A_0e^{-0.0001210t} = 0.71A_0$. Solving, we have $e^{-0.0001210t} = 0.71$, giving $-0.0001210t = \ln 0.71$, and finally $t \approx 2830$. That is, the artifact was made around 2830 years ago.

6 Solution is $(-6, -2)$.

7 From 2nd equation: $z = y - 1$. Substitute $y - 1$ for z in the 3rd equation, so that the 1st and 3rd equation give us the system

$$\begin{cases} x + y = -4 \\ 2x + 4y = -18 \end{cases}$$

This solves to give $x = 1$ and $y = -5$. Then $z = -5 - 1 = -6$, and the solution is $(1, -5, -6)$.

8 Let x, y, and z be the number of pennies, nickels, and dimes, respectively. Then we have $0.01x + 0.05y + 0.10z = 8.40$, $z = 2x - 6$, and $y = z$. This system solves to give $x = 30$, $y = 54$, $z = 54$.