Math 120 Exam #3 Key (Spring 2024)

1 Distance: $\sqrt{(-2-8)^2+(5-(-1))^2}=\sqrt{100+36}=\sqrt{136}=2\sqrt{34}$. Midpoint:

$$\left(\frac{-2+8}{2},\frac{5+(-1)}{2}\right)=(3,2).$$

2 Complete the square:

$$(x^2+10x+25)+(y^2-4y+4)=20+25+4 \quad\hookrightarrow\quad (x+5)^2+(y-2)^2=49.$$

Center is $(-5,2)$, radius is 7.

3 With $-\frac{b}{2a}=1$, vertex is at $(1,f(1))=(1,-16)$. Domain is $(-\infty,\infty)$ and range is $[-16,\infty)$.

4 Consider a rectangle with length x and width y, so perimeter is $2x+2y$. It must be that $2x+2y=80$, so that $y=40-x$. Area A of the rectangle is $A=xy$, or, as a function of x alone, $A(x)=x(40-x)=-x^2+40x$. This is a quadratic function with coefficients $a=-1$ and $b=40$. Since $-\frac{b}{2a}=20$, the vertex of the parabola is at $(20,A(20))=(20,400)$, which is the highest point on the parabola. Thus the area of the rectangle is maximized if $x=20$ and $y=20$, so that the rectangle is 20 m $\times$ 20 m and has area 400 m^2.

5 From the long division

$$\begin{array}{r|rrrrr}
 & & & x^2 & +4x & +7 \\ \hline
x^2-2x+1 & x^4 & +2x^3 & & -9x & -16 \\
 & -x^4 & +2x^3 & -x^2 & & \\ \hline
 & & 4x^3 & -x^2 & -9x & \\
 & & -4x^3 & +8x^2 & -4x & \\ \hline
 & & & 7x^2 & -13x & -16 \\
 & & & -7x^2 & +14x & -7 \\ \hline
 & & & & x & -23
\end{array}$$

we have

$$\frac{x^4+2x^3-9x-16}{x^2-2x+1}=x^2+4x+7+\frac{x-23}{x^2-2x+1}.$$

6 The model is $f(x)=C(x+3)[x-(2+i)][x-(2-i)]$, where $2-i$ must also be a zero in order to have real coefficients. Expanding yields $f(x)=C(x^3-x^2-7x+15)$, so that $f(1)=8C$. To satisfy $f(1)=10$ we must have $8C=10$, or $C=\frac{5}{4}$. Therefore

$$f(x)=\frac{5}{4}x^3-\frac{5}{4}x^2-\frac{35}{4}x+\frac{75}{4}.$$

7 Setting $f(x)=x^4-3x^3-20x^2-24x-8$, the equation is $f(x)=0$. Possible rational zeros of f are $\pm1,\pm2,\pm4,\pm8$. Through trial-and-error we find -1 is a zero of f, so that $x+1$ is a factor of $f(x)$, and with synthetic division we obtain $f(x) \div (x+1) = x^3 - 4x^2 - 16x - 8$. Hence

$$f(x) = (x+1)(x^3 - 4x^2 - 16x - 8).$$

Let $g(x) = x^3 - 4x^2 - 16x - 8$, which has possible rational zeros $\pm 1, \pm 2, \pm 4, \pm 8$. Again we apply trial-and-error, and find that -2 is a zero for g (and hence f) by obtaining a remainder of 0 when dividing $g(x)$ by $x+2$. We have $g(x) \div (x+2) = x^2 - 6x - 4$, and so

$$g(x) = (x+2)(x^2 - 6x - 4).$$

Since $f(x) = (x+1)g(x)$, we now have

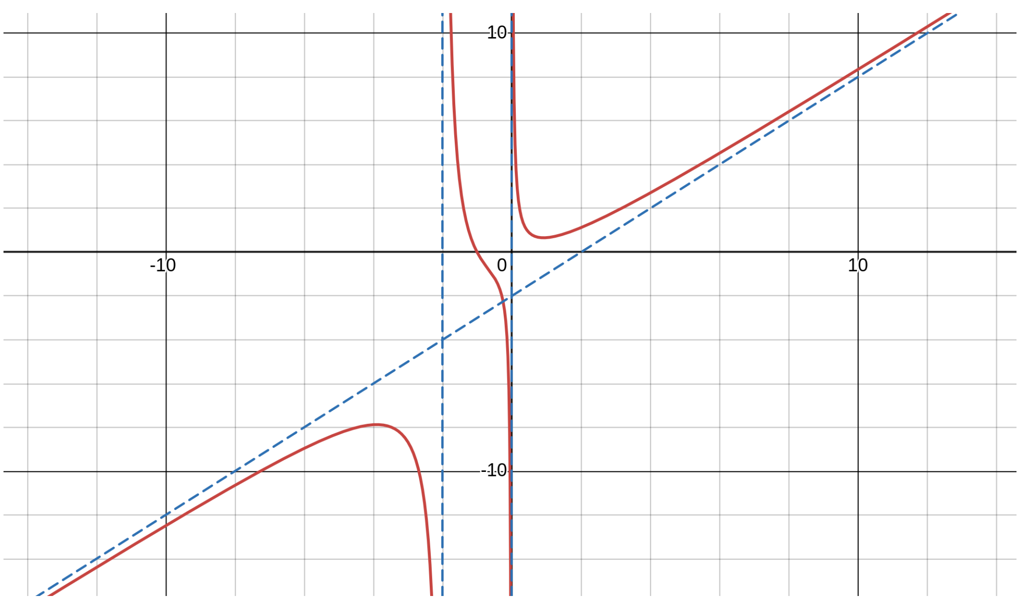
$$f(x) = (x+1)(x+2)(x^2 - 6x - 4).$$

To satisfy $f(x) = 0$ we may have $x = -1$, $x = -2$, or $x^2 - 6x - 4 = 0$. The solutions to the last equation are

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-4)}}{2(1)} = 3 \pm \sqrt{13},$$

which are two more zeros for f. The solutions to $f(x) = 0$ are: $\{-2, -1, 3 - \sqrt{13}, 3 + \sqrt{13}\}$.

8 (1) $D_R = \{x \,|\, x \neq -2, 0\}$; (2) No symmetry; (3) x-intercept is -1, no y-intercept; (4) vertical asymptotes are $x = -2$, $x = 0$; (5) slant asymptote is $y = x - 2$; (6) It's helpful to get, say, $R(-3) \approx -8.67$ and $R(4) \approx 2.71$ to fully ascertain where the graph lies above the x-axis or below it. For (7) the sketch should resemble the graph below.



9a Factor by grouping: $x^2(x+1) + 4(x+1) > 0$ yields $(x+1)(x^2+4) > 0$. This can be solved by the usual method using the Intermediate Value Theorem (IVT); however, we note here that $x^2 + 4 > 0$ holds for any real x, so it's only required that $x + 1 > 0$ hold to satisfy the inequality. Solution set is $(-1, \infty)$.

9b Get 0 on one side and a single quotient on the other:

$$\frac{x-2}{x+2} - 2 \leq 0 \quad \hookrightarrow \quad -\frac{x+6}{x+2} \leq 0 \quad \hookrightarrow \quad \frac{x+6}{x+2} \geq 0.$$

Let $f(x) = \frac{x+6}{x+2}$, so inequality is $f(x) \geq 0$. Now, $f(x) = 0$ only if $x = -6$, and $f(x)$ is undefined only if $x = -2$. Use -6 and -2 to partition the real line into subintervals $(-\infty, -6)$, $(-6, -2)$, and $(-2, \infty)$. Pick a test value in each subinterval to find where $f(x) > 0$ using the IVT. Knowing where $f(x) > 0$ and where $f(x) = 0$ solves $f(x) \geq 0$. Solution set is $(-\infty, -6] \cup (-2, \infty)$.