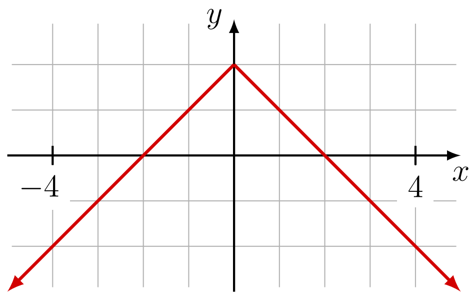
Math 120 Exam #1 Key (Spring 2024)

1a Quadrants I, IV.

1b Quadrants II, IV.

2 Make a table of values to obtain the graph below.



3a Get $7x - 2 = 4x - 5$, and then $x = -1$.

3b Multiply by $(x+3)(x-2)$ to get

$$6(x-2) + 20 = 5(x+3) \quad \Rightarrow \quad x = 7.$$

Solution set is $\{7\}$.

4 Let x be the number of times the bridge is crossed. With the transponder the cost is $27.50 + 5x$, and without it the cost is $7.50x$. We find x such that $27.50 + 5x = 7.50x$, which solves to give $x = 11$ bridge crossings.

5 Get $I(R+r) = E$, and so $I = \dfrac{E}{R+r}$.

6a FOIL yields $18 - 27i + 4i^2 = 14 - 27i$.

6b $\dfrac{4i}{2+i} \cdot \dfrac{2-i}{2-i} = \dfrac{8i - 4i^2}{4 - i^2} = \dfrac{4+8i}{5} = \dfrac{4}{5} + \dfrac{8}{5}i.$

7 See the long division work below. Since the remainder is 1, we have $i^{833} = i^1 = i$.

$$\begin{array}{r} 208 \\ 4\,\overline{)833} \\ 8 \\ \hline 033 \\ 32 \\ \hline 1 \end{array}$$

8a $x = \dfrac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(1)}}{2(3)} = \dfrac{6 \pm 2\sqrt{6}}{6} = \dfrac{3 \pm \sqrt{6}}{3}$.

8b $x^2 + 6x = 5 \hookrightarrow x^2 + 6x + 9 = 5 + 9 \hookrightarrow (x+3)^2 = 14 \hookrightarrow x + 3 = \pm\sqrt{14}$, and so we obtain $x = -3 \pm \sqrt{14}$.

9 Let x be the width of the lot, so the length is $x + 3$. Since the area is 180 m^2, we must have $x(x+3) = 180$, or $x^2 + 3x - 180 = 0$. Factoring, we get $(x+15)(x-12) = 0$, which has solutions $x = -15, 12$. Thus the width must be 12 m and the length 15 m; that is, the lot is 12 m $\times$ 15 m.

10a Write $\sqrt{x-4} = 5 - \sqrt{x+1}$. Square: $x - 4 = 25 - 10\sqrt{x+1} + (x+1)$. Isolate radical: $\sqrt{x+1} = 3$. Square again: $x + 1 = 9$, and thus $x = 8$. This solution is valid. Solution set: $\{8\}$.

10b Make the substitution $u = x^{1/4}$ to get $u^2 + 3u - 10 = 0$, and thus either $u = -5$ or $u = 2$. That is, $x^{1/4} = -5$ or $x^{1/4} = 2$. Since $x^{1/4} = \sqrt[4]{x} = -5$ is impossible, the only solution obtains from $x^{1/4} = 2$, which yields $x = 16$. Solution set: $\{16\}$.

10c We get $|x - 3| = 8 \;\Rightarrow\; x - 3 = \pm 8 \;\Rightarrow\; x = 3 \pm 8$. Solution set: $\{-5, 11\}$.

11a $6x - 9 \geq -4x - 3 \;\Rightarrow\; 10x \geq 6 \;\Rightarrow\; x \geq \frac{3}{5}$. Solution set: $\left[\frac{3}{5}, \infty\right)$.

11b Either $\frac{2x+6}{3} > 2$ or $\frac{2x+6}{3} < -2$, so that $2x + 6 > 6$ or $2x + 6 < -6$, and finally $x > 0$ or $x < -6$. Solution set is $(-\infty, -6) \cup (0, \infty)$.