Math 120 Exam #4 Key (Spring 2023)

1a $D_f = \{x \mid 8x+3>0\} = (-\frac{3}{8}, \infty)$.

1b $D_f = \left\{x \;\middle|\; \dfrac{2x-4}{x^2-4} > 0\right\} = \left\{x \;\middle|\; \dfrac{2}{x+2} > 0 \text{ and } x \neq \pm 2\right\} = (-2,2)\cup(2,\infty)$.

2 $\log \dfrac{x(x^2-1)}{7(x+1)} = \log \dfrac{x(x-1)}{7}$.

3 $\log_b \sqrt[3]{\dfrac{25}{16}} = \dfrac{1}{3}\log_b\left(\dfrac{25}{16}\right) = \dfrac{2}{3}\log_b\left(\dfrac{5}{4}\right) = \dfrac{2}{3}(\log_b 5 - 2\log_b 2) = \dfrac{2}{3}(\beta - 2\alpha)$.

4a Get $8^{1-2x} = 8^{2(x-4)}$, which implies $1-2x = 2(x-4)$, and so $x = \frac{9}{4}$.

4b Let $u = 2^x$ to get $u^2+u-12=0$, and thus $u=-4$ or $u=3$. Now, $2^x=-4$ has no solution, but $2^x = 3$ gives $x = \frac{\ln 3}{\ln 2}$.

4c Write $\log_3(x+6)(x+4) = 1$, so $(x+6)(x+4)=3$ and hence $x=-7,-3$. But $x=-7$ is extraneous. Solution set is $\{-3\}$.

4d $|\ln x| = 4$ implies $\ln x = \pm 4$, and hence $x = e^{\pm 4}$.

5 For $A(t) = A_0e^{-kt}$ we have $\frac{1}{2}A_0 = A(7340) = A_0e^{-7340k}$, so $e^{-7340k} = \frac{1}{2}$, and hence $k = 0.00009443$. The completed model is now $A(t) = A_0e^{-0.00009443t}$, and we find t such that $A(t) = 0.01A_0$. This implies

$$A_0e^{-0.00009443t} = 0.01A_0,$$

or $e^{-0.00009443t} = 0.01$. Solving, we get $t \approx 48{,}768$ years.

6 Growth rate is 0.82%. As for the doubling time, it is given by $(\ln 2)/k = (\ln 2)/0.0082 = 84.5$ years.

7 Solution is $(-6,-2)$.

8 Note from the first two equations that $x+y+6z = 3 = x+y+3z$, so $6z=3z$, and hence $z=0$. This immediately reduces the system to two variables, and solving yields $x=-1$ and $y=4$. Solution is $(-1,4,0)$.

9 Letting x be the number of rooms with a kitchen, and y the number without a kitchen. We obtain the following system:

$$\begin{cases} x + y = 200 \\ 200x + 160y = 34{,}000 \end{cases}$$

Solving yields $x = 50$ and $y = 150$.