Math 120 Exam #3 Key (Spring 2023)

1 Complete squares to get $(x-2)^2+(y+1)^2=9$. Center is at $(2,-1)$, radius is 3.

2a It's at the midpoint between the given points, so use the midpoint formula to get $(4,5)$.

2b It's the distance between the center $(4,5)$ and either of the given points. Use the distance formula to get $r=\sqrt{2}$.

2c With center $(4,5)$ and radius $\sqrt{2}$ the equation is $(x-4)^2+(y-5)^2=2$.

3 With $-\frac{b}{2a}=\frac{5}{4}$, vertex is at $(\frac{5}{4}, f(\frac{5}{4}))=(\frac{5}{4},-\frac{73}{8})$. The domain is $(-\infty,\infty)$ and the range is $[-\frac{73}{8},\infty)$.

4 Have $f(x)=a(x-h)^2+k$ with $(h,k)=(-3,-1)$, so $f(x)=a(x+3)^2-1$. Now use the fact that $f(-2)=-3$ to find that $a=-2$. Therefore $f(x)=-2(x+3)^2-1$.

5 From the long division

$$\begin{array}{r|l}
 & \phantom{x^4+{}}x^2 \qquad\quad\ \ +4 \\ \hline
x^2-2 & x^4+2x^2-5x-16 \\
 & -x^4+2x^2 \\ \cline{2-2}
 & \phantom{x^4+{}}4x^2-5x-16 \\
 & -4x^2 \qquad\ \ +8 \\ \cline{2-2}
 & -5x\ \ -8
\end{array}$$

we have

$$\frac{x^4+2x^3-4x^2-5x-6}{x^2-2}=x^2+4-\frac{5x+8}{x^2-2}.$$

6 The model is $f(x)=C(x-2)[x-(2-3i)][x-(2+3i)]$, where $2+3i$ must also be a zero in order to have real coefficients. Expanding yields $f(x)=C(x^3-6x^2+21x-26)$, and to satisfy $f(1)=-10$ we must have $C=1$. Therefore

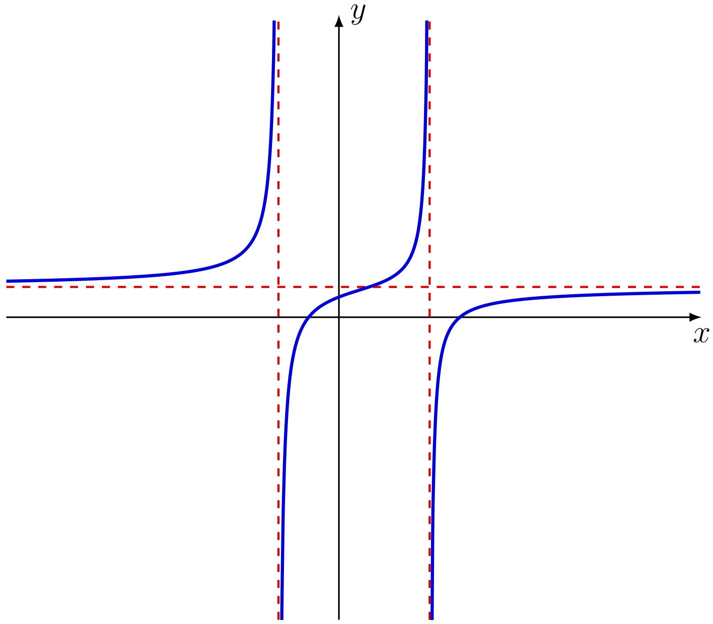
$$f(x)=x^3-6x^2+21x-26.$$

7 With $f(x)=x^4-x^3+2x^2-4x-8$, equation is $f(x)=0$. Possible rational zeros of f are $\pm1,\pm2,\pm4,\pm8$. Through trial-and-error we find 2 is a zero of f, so that $x-2$ is a factor of $f(x)$, and with synthetic division we obtain $f(x)\div(x-2)=x^3+x^2+4x+4$. Now

$$f(x)=(x-2)(x^3+x^2+4x+4)=(x-2)[x^2(x+1)+4(x+1)]=(x-2)(x+1)(x^2+4).$$

From this factorization we obtain the zeros of f, which are also the solutions to the given equation: $\{2,-1,2i,-2i\}$.

8 (1) $D_R = \{x \,|\, x \neq -2, 3\}$; (2) No symmetry; (3) x-intercepts are $-1, 4$, and y-intercept is $R(0) = \frac{2}{3}$; (4) v.a. are $x = -2$ and $x = 3$; (5) h.a. is $y = 1$; (6) It's helpful to get, say, $R(-3) = \frac{7}{3}$ and perhaps $R(5) = \frac{3}{7}$. For (7) the sketch should resemble the graph below.



9a Solution set is $(-4, -\frac{1}{2})$.

9b Solution set is $(-\infty, -2) \cup [6, \infty)$.