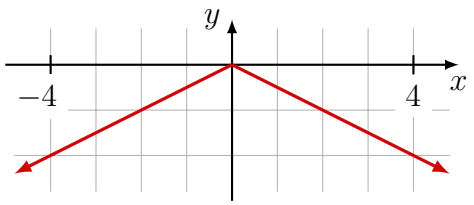
Math 120 Exam #1 Key (Spring 2023)

1a Quadrants II, IV.

1b No quadrant works.

2



3a Get $7x - 2 = 4x - 5$, and then $x = -1$.

3b Multiply by $(x+3)(x-2)$ to get

$$6(x-2) - 5(x+3) = -20 \quad \Rightarrow \quad x = 7.$$

Solution set is $\{7\}$.

4 Let x be the amount invested at 4%, so $\$4000 - x$ is the amount invested at -3%. Then

$$0.04x - 0.03(4000 - x) = 55,$$

which solves to give $x = \$2500$. So \$2500 was invested at 4% and \$1500 was invested at -3%.

5 After showing some work, you should get $h = \dfrac{A - 2\ell w}{2\ell + 2w}$.

6a FOIL procedure gives $25 - 20i + 4i^2 = 21 - 20i$.

6b $\dfrac{4+i}{2-i} \cdot \dfrac{2+i}{2+i} = \dfrac{8 + 6i + i^2}{4 - i^2} = \dfrac{7}{5} + \dfrac{6}{5}i.$

7 The division $877/4$ has remainder 1, and so $i^{877} = i^1 = i$.

8a Get $(x-6)^2 = 49$, so $x - 6 = \pm 7$, and finally $x = -1, 13$.

8b We get $x^2 + 4x = -\frac{1}{2}$, then $x^2 + 4x + 4 = -\frac{1}{2} + 4$, and then $(x+2)^2 = \frac{7}{2}$. Solutions are $x = \pm\sqrt{\frac{7}{2}} - 2$.

9 Let x be the length of one piece, which makes a square $x/4$ meters to a side. The other length is $8-x$, which makes a square $\frac{8-x}{4}$ to a side. The areas of the squares is $(x/4)^2$ and $\left(\frac{8-x}{4}\right)^2$, and we're given that

$$\left(\frac{x}{4}\right)^2 + \left(\frac{8-x}{4}\right)^2 = 2.$$

Solving yields $x = 4$, so each piece of wire is 4 meters long.

10a Write $\sqrt{2x-3} = 1 + \sqrt{x-2}$, square to get $2x - 3 = 1 + 2\sqrt{x-2} + (x-2)$, and then isolate the remaining radical to get

$$2\sqrt{x-2} = x - 2 \quad \Rightarrow \quad 4(x-2) = (x-2)^2 \quad \Rightarrow \quad x^2 - 8x + 12 = 0.$$

The trinomial factors, giving $(x-6)(x-2) = 0$, and therefore $x = 2, 6$.

10b Factor: $(2x^{1/3} - 3)(x^{1/3} + 5) = 0$, so $2x^{1/3} = 3$ or $x^{1/3} = -5$, and hence $x = \frac{27}{8}, -125$. (The substitution $u = x^{1/3}$ may help but is not essential.)

10c We get $|2x - 1| = 9$, and hence $2x - 1 = \pm 9$. Solutions are $x = -4, 5$.

11a Solving leads to $x \geq 8$, so the solution set is $[8, \infty)$.

11b We get $|-2x + 7| > 4$, implying $-2x + 7 > 4$ or $-2x + 7 < -4$, and thus $x < \frac{3}{2}$ or $x > \frac{11}{2}$. Solution set is $(-\infty, \frac{3}{2}) \cup (\frac{11}{2}, \infty)$.

11c Divide by -3 to get $|x+7| \leq 9$, so $-9 \leq x+7 \leq 9$, and therefore $-16 \leq x \leq 2$. Solution set is $[-16, 2]$.