Math 120 Exam #4 Key (Spring 2022)

1 $A = 6000(1 + 0.0088/4)^{16} = \$6214.72$, $A = 6000e^{0.0084(4)} = \$6205.03$. The quarterly compounded investment reaps the greater reward.

2a $(-14, \infty)$

2b Need $x^2 - 4x - 12 > 0$, so domain is $(-\infty, -2) \cup (6, \infty)$.

3 $\log_4 \dfrac{x^{1/3}(x+1)^2}{y^{1/3}}$.

4 $C - 2A$

5a $5^{2-x} = 5^{-3}$ implies $2 - x = -3$, so $x = 5$.

5b Let $u = e^x$, so $u^2 - 2u - 3 = 0$, giving $e^x = u = -1, 3$. But $e^x = -1$ has no solution, leaving $e^x = 3$ to yield the final solution $x = \ln 3$.

5c $8^2 = 1 - 3x$, giving $x = -21$.

5d Write $\log_3 \dfrac{(x+4)^2}{9} = 2$, so $\dfrac{(x+4)^2}{9} = 3^2$, giving $x = -13, 5$. But -13 is extraneous, so solution set is $\{5\}$.

6 For $A(t) = A_0 e^{-kt}$ we have $\frac{1}{2}A_0 = A(7340) = A_0 e^{-7340k}$, so $e^{-7340k} = \frac{1}{2}$, and hence $k = 0.00009443$. The completed model is now $A(t) = A_0 e^{-0.00009443t}$, and we find t such that $A(t) = 0.18A_0$. This implies

$$A_0 e^{-0.00009443t} = 0.18A_0,$$

or $e^{-0.00009443t} = 0.18$. Solving, we get $t \approx 18{,}159$ years.

7 Solution is $(-6, -2)$.

8 Letting x be the first number and y the second number, we obtain the system

$$\begin{cases} 3x + 2y = 8 \\ 2x - y = 3 \end{cases}$$

Solving yields $x = 2$ and $y = 1$.

9 Solution is $(0, 1, 2)$.

10 Solution set is $\big\{(0, -2), (0, 2), \big(-1, -\sqrt{3}\big), \big(-1, \sqrt{3}\big)\big\}$.

11 Let x, y, and z be the number of \$8, \$10, and \$12 tickets sold, respectively. Then we obtain the system

$$\begin{cases} x + y + z = 400 \\ 8x + 10y + 12z = 3700 \\ x + y = 7z \end{cases}$$

Solving the system yields $(x, y, z) = (200, 150, 50)$.