Math 120 Exam #3 Key (Spring 2022)

1 $\sqrt{(-\frac{1}{2}+5)^2+(3-7)^2} = \sqrt{145/4} = \dfrac{\sqrt{145}}{2}$.

2 Complete squares to get $(x-2)^2+(y-6)^2 = 49$. Center is at $(2,6)$, radius is 7.

3 With $-\frac{b}{2a} = -3$, vertex is at $(-3, f(-3)) = (-3, 21)$. The domain is $(-\infty,\infty)$ and the range is $(-\infty, 21]$.

4a Rocket reaches maximum height at time $t = -\frac{b}{2a} = 4$ seconds.

4b Maximum height is $h(4) = 256$ feet.

4c Find times t for which $h(t) = 0$, or $-16t^2 + 128t = 0$. Solutions are $t = 0$ and $t = 8$. So rocket returns to Earth at 8 seconds.

5 From the long division

$$
\begin{array}{r l}
 & x^2 \quad + x \quad - 3 \\
x^2+x-2 \,\big)\!\! & x^4 + 2x^3 - 4x^2 - 5x \quad - 6 \\
 & -x^4 \quad - x^3 + 2x^2 \\
 & \qquad\quad x^3 - 2x^2 - 5x \\
 & \qquad\quad -x^3 \quad - x^2 + 2x \\
 & \qquad\qquad\quad -3x^2 - 3x \quad - 6 \\
 & \qquad\qquad\qquad 3x^2 + 3x \quad - 6 \\
 & \qquad\qquad\qquad\qquad\qquad - 12
\end{array}
$$

we have

$$\frac{x^4+2x^3-4x^2-5x-6}{x^2+x-2} = x^2+x-3-\frac{12}{x^2+x-2}.$$

6 Equation is $f(x) = 0$ with $f(x) = x^4 - 2x^2 - 16x - 15$. Possible rational zeros are $\pm1, \pm3, \pm5, \pm15$. Through trial-and error we find -1 is a hit:

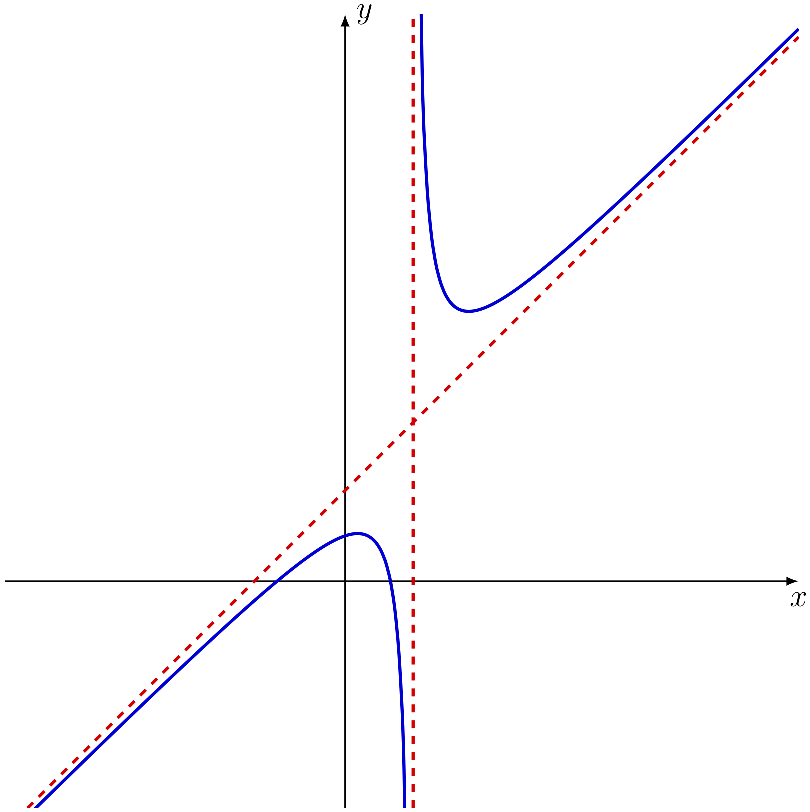
$$
\begin{array}{r|rrrr|r}
-1 & 1 & 0 & -2 & -16 & -15 \\
 & & -1 & 1 & 1 & 15 \\
\hline
 & 1 & -1 & -1 & -15 & 0
\end{array}
\quad \longrightarrow \quad f(x) = (x+1)(x^3-x^2-x-15).
$$

The factor $g(x) = x^3 - x^2 - x - 15$ has the same list of possible rational zeros, and we find 3 is one:

$$
\begin{array}{r|rrr|r}
3 & 1 & -1 & -1 & -15 \\
 & & 3 & 6 & 15 \\
\hline
 & 1 & 2 & 5 & 0
\end{array}
\quad \longrightarrow \quad g(x) = (x-3)(x^2+2x+5).
$$

Now $f(x) = (x+1)(x-3)(x^2+2x+5)$, so $f(x) = 0$ if and only if $x = -1, 3, -1 \pm 2i$.

7 (1) No symmetry; (2) $R(0) = 2$; (3) $R(x) = 0$ if $x = -3, 2$; (4) v.a. is $x = 3$; (5) $y = x+4$ is slant asymptote; (6) additional points as needed. For (7) the sketch should resemble the graph below.



8a Let $f(x) = (x+1)(x-2)(x+3)$, so inequality is $f(x) > 0$. The x-intercepts are $-3, -1, 2$. For intervals $(-\infty, -3)$, $(-3, -1)$, $(-1, 2)$, $(2, \infty)$ choose test values like -4, -2, 0, 3, respectively. Since $f(-4) < 0$, $f(-2) > 0$, $f(0) < 0$, $f(3) > 0$, by the Intermediate Value Theorem we conclude that $f(x) > 0$ on $(-3, -1) \cup (2, \infty)$.

8b Write as

$$\frac{x}{x-6} - 1 \leq 0 \quad \hookrightarrow \quad \frac{6}{x-6} \leq 0.$$

Now, $\frac{6}{x-6} = 0$ has no solution, while $\frac{6}{x-6} < 0$ can only occur if $x - 6 < 0$, or $x < 6$. Solution set is therefore $(-\infty, 6)$.