Math 120 Exam #2 Key (Spring 2022)

1 $f(-1) = 12$, $f(-x) = f(x+3) = x^2 + 3x + 8$.

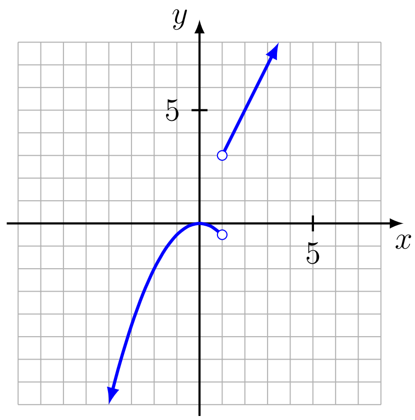
2a $g(-1) = 0$, $g(0) = 2$, $g(3)$ is undefined.

2b $\operatorname{Dom} g = [-3, 3)$, $\operatorname{Ran} g = [0, 3)$.

3a Symmetric about origin only.

3b Neither.

4a



4b The graph may help: $\operatorname{Dom} p = (-\infty, 1) \cup (1, \infty)$, $\operatorname{Ran} p = (-\infty, 0] \cup (3, \infty)$.

5 Slope is $-\frac{5}{6}$, so $y - (-2) = -\frac{5}{6}(x - (-3))$ is the equation, which becomes $y = -\frac{5}{6}x - \frac{9}{2}$.

6 $x = -3$

7 Equation is $y - (-6) = -\frac{3}{2}(x - 2)$, which in slope-intercept form is $y = -\frac{3}{2}x - 3$. The y-intercept is -3.

8 $y - 2x + 5 = 0$ becomes $y = 2x - 5$, so the given line has slope 2, and hence L has slope $-\frac{1}{2}$. Equation for L is thus $y - 2 = -\frac{1}{2}(x + 1)$, or $y = -\frac{1}{2}x + \frac{3}{2}$.

9a $\operatorname{Dom} f = (-\infty, -2) \cup (-2, 2) \cup (2, \infty)$.

9b $\operatorname{Dom} r = \{x \mid x \neq 0 \text{ and } 12/x \neq 4\} = (-\infty, 0) \cup (0, 3) \cup (3, \infty)$.

10a $\operatorname{Dom} F = [-8, \infty)$, $\operatorname{Dom} G = (-\infty, 10]$.

10b $(F - G)(x) = \sqrt{x+8} - \sqrt{10-x}$ with $\operatorname{Dom}(F-G) = \operatorname{Dom} F \cap \operatorname{Dom} G = [-8, 10]$.

10c $(F/G)(x) = \dfrac{\sqrt{x+8}}{\sqrt{10-x}}$ with $\operatorname{Dom}(F/G) = [-8, 10)$.

11a $(f \circ g)(x) = f(g(x)) = f(\frac{1}{2x}) = \dfrac{5}{\frac{1}{2x} - 4}$.

11b $\operatorname{Dom} f = \{x \,|\, x \neq 4\}$ and $\operatorname{Dom} g = \{x \,|\, x \neq 0\}$, so

$$\begin{aligned}
\operatorname{Dom}(f \circ g) &= \{x \,|\, x \in \operatorname{Dom} g \text{ and } g(x) \in \operatorname{Dom} f\} \\
&= \{x \,|\, x \neq 0 \text{ and } \tfrac{1}{2x} \neq 4\} \\
&= \{x \,|\, x \neq \tfrac{1}{8}, 0\} \\
&= (-\infty, 0) \cup (0, \tfrac{1}{8}) \cup (\tfrac{1}{8}, \infty).
\end{aligned}$$

12a Set $y = f(x)$, and solve for x:

$$y = \frac{2x+1}{6-x} \quad \hookrightarrow \quad 6y - xy = 2x + 1 \quad \hookrightarrow \quad x = \frac{6y-1}{y+2} \quad \hookrightarrow \quad f^{-1}(y) = \frac{6y-1}{y+2}.$$

12b $\operatorname{Ran} f^{-1} = \operatorname{Dom} f = (-\infty, 6) \cup (6, \infty)$ and $\operatorname{Ran} f = \operatorname{Dom} f^{-1} = (-\infty, -2) \cup (-2, \infty)$.