MATH 120 EXAM #4 KEY (SPRING 2019)

1a Suppose $f(a) = f(b)$, giving

$$\frac{6a-3}{2a+1} = \frac{6b-3}{2b+1} \quad \Rightarrow \quad (6a-3)(2b+1) = (6b-3)(2a+1) \quad \Rightarrow \quad 12a = 12b \quad \Rightarrow \quad a = b,$$

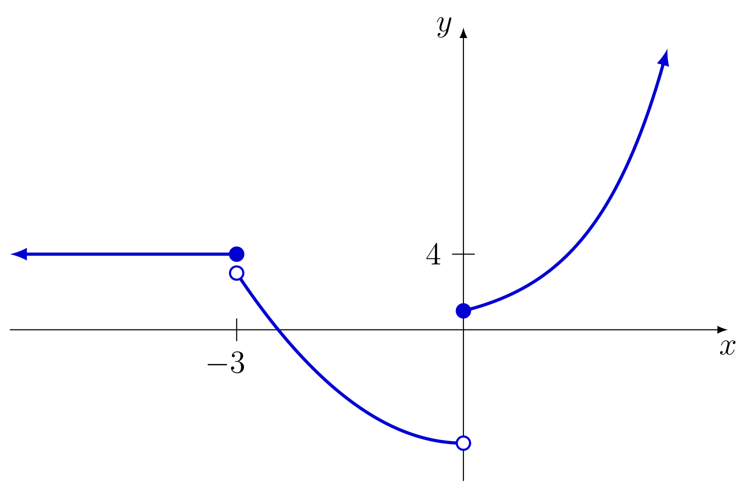
and therefore f is one-to-one.

1b By definition $f(x) = y$ if and only if $f^{-1}(y) = x$. Let $y = f(x)$, so $y = (6x-3)/(2x+1)$. Solving for x gives $x = (y+3)/(6-2y)$, and hence

$$f^{-1}(y) = \frac{y+3}{6-2y}.$$

2 $y = x^2 - 2x$

3



4 $e^t = W^5$ and $\log 2 = 0.3010$.

5 $(-\infty, 0) \cup (0, \infty)$

6 With laws of logarithms,

$$3\log_a x - \tfrac{5}{2}\log_a p - 4\log_a q.$$

7 $\frac{1}{3}$

8a We get $3^3 = 3^{5x+2x^2}$, implying $2x^2 + 5x = 3$, and hence $x = \frac{1}{2}, -3$.

8b Write as $e^{2x} - 5e^x + 1 = 0$, and so by the quadratic formula

$$e^x = \frac{5 \pm \sqrt{21}}{2},$$

and therefore

$$x = \ln\left(\frac{5 \pm \sqrt{21}}{2}\right).$$

8c Rewrite as $2^5 = 10 + 3x$, and hence $x = \frac{22}{3}$.

8d We have

$$\log_4(x^2 - 9) = 2 \quad \Rightarrow \quad 4^2 = x^2 - 9 \quad \Rightarrow \quad x = \pm 5.$$

But $x = -5$ is extraneous, so the solution set is $\{5\}$.

8e Square both sides to get

$$\ln x = \left(\ln \sqrt[4]{x}\right)^2 = \left(\tfrac{1}{4}\ln x\right)^2 = \tfrac{1}{16}(\ln x)^2,$$

and hence $(\ln x)^2 - 16\ln x = 0$. Factoring gives

$$(\ln x)(\ln x - 16) = 0,$$

so that either $\ln x = 0$ (so $x = 1$) or $\ln x = 16$ (so $x = e^{16}$). Solution set is $\{1, e^{16}\}$.

9 The basic model is $A(t) = A_0 e^{-kt}$, but we know $A(5750) = A_0/2$; that is,

$$A_0 e^{-5750k} = \frac{A_0}{2} \quad \Rightarrow \quad e^{-5750k} = \frac{1}{2} \quad \Rightarrow \quad k = \frac{\ln 2}{5750} \approx 0.0001205.$$

Now we have $A(t) = A_0 e^{-0.0001205t}$, and we must find t such that $A(t) = 0.667A_0$:

$$A_0 e^{-0.0001205t} = 0.667A_0 \quad \Rightarrow \quad e^{-0.0001205t} = 0.667 \quad \Rightarrow \quad -0.0001205t = \ln 0.667,$$

and so

$$t = \frac{\ln 0.667}{-0.0001205} \approx 3361 \text{ years}$$

is the age of the mummies at the time of discovery.

10 Solution is $\left(\frac{1}{2}, \frac{3}{4}\right)$.

11 System is consistent but dependent, with solution set expressible as

$$\left\{(x, 5x - 12, 11x - 28) : x \text{ is real}\right\}.$$

12 Let x be the number of pounds of French roast, and y the number of pounds of Colombian. Noting that $10.50(20) = 210$, we have

$$\begin{cases} x + y = 20 \\ 12x + 9.50y = 210 \end{cases}$$

Solving the system gives $x = 8$ and $y = 12$.