Math 120 Exam #3 Key (Spring 2019)

1 Zeros are 0 (multiplicity 1), 19 (multiplicity 9), and -17 (multiplicity 4).

2 We have $f(-3) = 50$ and $f(1) = -2$, so 0 lies between $f(-3)$ and $f(1)$, and therefore there exists some $c \in (-3, 1)$ such that $f(c) = 0$ by the Intermediate Value Theorem.

3 From the long division

$$\begin{array}{r|rrrrr}
 & & x^3 & -2x^2 & +2x & -4 \\ \hline
x+2 & x^4 & & -2x^2 & & +3 \\
 & -x^4 & -2x^3 & & & \\ \hline
 & & -2x^3 & -2x^2 & & \\
 & & 2x^3 & +4x^2 & & \\ \hline
 & & & 2x^2 & & \\
 & & & -2x^2 & -4x & \\ \hline
 & & & & -4x & +3 \\
 & & & & 4x & +8 \\ \hline
 & & & & & 11
\end{array}$$

we see $Q(x) = x^3 - 2x^2 + 2x - 4$ and $R(x) = 11$, and thus

$$P(x) = (x + 2)(x^3 - 2x^2 + 2x - 4) + 11.$$

4 Synthetic division shows the quotient is $4x^4 - 4x^3 + 4x^2 - 6x + 6$ and the remainder is -11:

$$\begin{array}{r|rrrrr|r}
-1 & 4 & 0 & 0 & -2 & 0 & -5 \\
 & & -4 & 4 & -4 & 6 & -6 \\ \hline
 & 4 & -4 & 4 & -6 & 6 & -11
\end{array}
\quad \longrightarrow \quad 4x^4 - 4x^3 + 4x^2 - 6x + 6 - \frac{11}{x+1}.$$

5 Possible rational zeros are $\pm1, \pm2, \pm3, \pm6, \pm9, \pm18, \pm27, \pm54$. Trial and error shows -2 is one:

$$\begin{array}{r|rrrr|r}
-2 & 1 & -7 & 9 & 27 & -54 \\
 & & -2 & 18 & -54 & 54 \\ \hline
 & 1 & -9 & 27 & -27 & 0
\end{array}$$

Thus, with factoring by grouping and the difference of cubes factoring formula, we obtain

$$\begin{aligned}
f(x) &= (x + 2)(x^3 - 9x^2 + 27x - 27) \\
&= (x + 2)[(x^3 - 27) - (9x^2 - 27x)] \\
&= (x + 2)[(x - 3)(x^2 + 3x + 9) - 9x(x - 3)] \\
&= (x + 2)(x - 3)(x^2 - 6x + 9) \\
&= (x + 2)(x - 3)^3.
\end{aligned}$$

The equation $f(x) = 0$ has solution set $\{-2, 3\}$.

6 The simplest choice is $x(x + 2)(x - 3)^2$, though $cx(x + 2)(x - 3)^2$ for any $c \neq 0$ works.

7 The additional zero $2+i$ will need to be included in order to have rational coefficients. Thus we have

$$f(x) = (x+1)[x-(2-i)][x-(2+i)] = (x+1)(x^2-4x+5) = x^3-3x^2+x+5.$$

8 Possible rational zeros are $\pm1, \pm2, \pm4, \pm8, \pm\frac{1}{2}$. Indeed -2 works:

$$\begin{array}{r|rrr|r} -2 & 2 & 7 & 2 & -8 \\ & & -4 & -6 & 8 \\ \hline & 2 & 3 & -4 & 0 \end{array}$$

So $f(x) = (x+2)(2x^2+3x-4)$. The trinomial $2x^2+3x-4$ does not lend itself to direct factoring. Setting $2x^2+3x-4=0$, the quadratic formula or completing the square gives $x = \frac{-3\pm\sqrt{41}}{4}$. Thus the zeros of f are -2, $\frac{-3+\sqrt{41}}{4}$, $\frac{-3-\sqrt{41}}{4}$, and $f(x)$ factors as

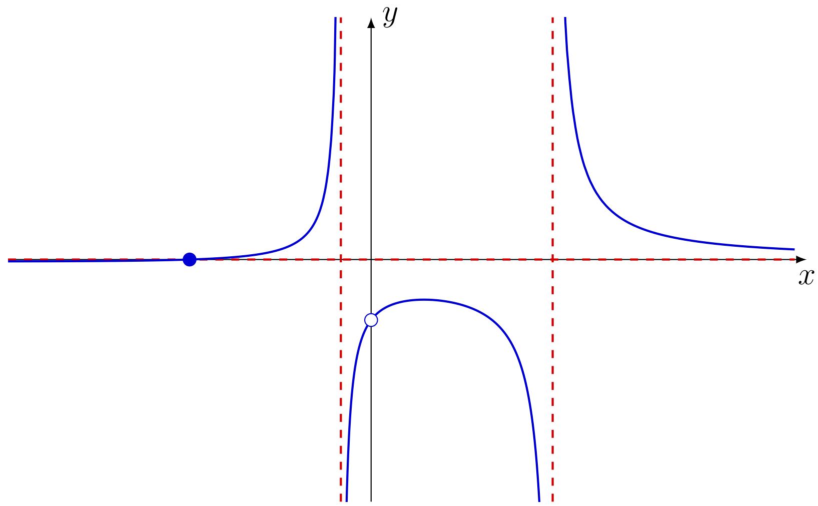
$$f(x) = 2(x+2)\left(x+\frac{3-\sqrt{41}}{4}\right)\left(x+\frac{3+\sqrt{41}}{4}\right).$$

Note the 2 at left of the factors: without it we would get the wrong lead coefficient!

9 Writing

$$G(x) = \frac{x(x+3)}{x(x-3)(2x+1)}$$

we see that $\text{Dom}(G) = \{x : x \neq -\frac{1}{2}, 0, 3\}$. Vertical asymptotes are $x=-\frac{1}{2}$ and $x=3$, with a hole at $(0,-1)$. Horizontal asymptote is $y=0$. The equation $G(x)=0$ yields only $x=-3$ as a solution, so $(-3,0)$ is the only intercept. Graph follows, with hole and intercept marked.



10a Factoring gives $(x-2)(x+1) > 0$, and hence either $x < -1$ or $x > 2$. Solution set is $(-\infty,-1)\cup(2,\infty)$.

10b Move everything to one side:

$$x^3+4x^2+x-6 \leq 0.$$

We find that 1 is a zero of the polynomial, and so with synthetic division we get

$$x^3 + 4x^2 + x - 6 = (x-1)(x^2 + 5x + 6) = (x-1)(x+2)(x+3) \leq 0.$$

Solution set: $(-\infty, -3] \cup [-2, 1]$.

10c We have

$$\frac{x}{x-2} + 1 \geq 0 \quad \Rightarrow \quad \frac{x-1}{x-2} \geq 0.$$

Solution set is $(-\infty, 1] \cup (2, \infty)$.