Math 120 Exam #2 Key (Spring 2019)

1a Replacing x with $-x$ does not result in the same equation, nor does replacing y with $-y$. Replacing both x and y with $-x$ and $-y$ also does not result in the same equation. The graph has no symmetry.

1b Since $3x = |-y|$ is the same as $3x = |y|$, the graph is symmetric about the x-axis.

2a $f(x) = \sqrt{x-7} - 4$

2b $f(x) = -(x+3)^2 + 2$

3a $2i \cdot 6i = 12i^2 = -12.$

3b $-6i + 24i^2 = -6i - 24 = -24 - 6i.$

3c $(5-4i)(1+2i) = 5 + 6i - 8i^2 = 13 + 6i.$

3d $\dfrac{i}{2+i} \cdot \dfrac{2-i}{2-i} = \dfrac{2i - i^2}{4 - i^2} = \dfrac{2i+1}{5} = \dfrac{1}{5} + \dfrac{2}{5}i.$

4 Factoring gives $t(3t-2)(t-1) = 0$, and so the solution set is $\{0, 1, \frac{2}{3}\}$.

5 We have

$$x^2 + 6x = -2 \quad \Rightarrow \quad x^2 + 6x + 9 = -2 + 9 \quad \Rightarrow \quad (x+3)^2 = 7 \quad \Rightarrow \quad x + 3 = \pm\sqrt{7},$$

and so the solution set is $\{-3 - \sqrt{7}, -3 + \sqrt{7}\}$.

6 The quadratic formula gives

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)} = \frac{1 \pm \sqrt{-3}}{2} = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i.$$

7 Factoring yields:

$$(y^2 - 16)(y^2 + 1) = 0,$$

so either $y^2 - 16 = 0$ (giving the solutions ± 4) or $y^2 + 1 = 0$ (giving the solutions $\pm i$). Solution set is $\{-4, 4, -i, i\}$.

8 Let ℓ be the length and w the width. We're given $2\ell + 2w = 28$ and $\ell w = 48$, which implies $(14 - w)w = 48$, and hence $(w-6)(w-8) = 0$. Thus we find that $w = 6, 8$. When $w = 6$ we get $\ell = 8$, and vice-versa. Therefore the dimensions are 6 ft by 8 ft.

9a Get into vertex form:

$$f(x) = -3(x^2 - 8x) - 49 = -3(x^2 - 8x + 16) - 49 - (-3)(16) = -3(x-4)^2 - 1.$$

So f has vertex at $(4, -1)$.

9b Maximum value is $f(4) = -1$.

9c Range: $(-\infty, -1]$.

9d Increasing on $(-\infty, 4)$, decreasing on $(4, \infty)$.

10 There are three sides of length x (perpendicular to the lake shore) and one side of length y (parallel to the lake shore). We have $3x + y = 240$, and so $y = 240 - 3x$. Area as a function of x is

$$A(x) = x(240 - 3x) = -3x^2 + 240x = -3(x^2 - 80x) = -3(x^2 - 80x + 1600) + 3(1600),$$

and thus

$$A(x) = -3(x - 40)^2 + 4800.$$

Area is maximized when $x = 40$. Largest total area is $A(40) = 4800$ m^2.

11a Since $x^3 + 27 = x^3 + 3^3 = (x + 3)(x^2 - 3x + 9)$, the equation simplifies to

$$18(x + 3) - x(x^2 - 3x + 9) = 81 \quad \Rightarrow \quad x^3 - 3x^2 - 9x + 27 = 0.$$

Factor by grouping:

$$x^2(x - 3) - 9(x - 3) = 0 \quad \Rightarrow \quad (x - 3)(x^2 - 9) = 0 \quad \Rightarrow \quad (x - 3)^2(x + 3) = 0.$$

Solutions to this last equation are ± 3; however, $x = -3$ is an extraneous solution. Solution set for the original equation is therefore $\{3\}$.

11b We have $\sqrt{a + 3} = 3$, and thus $a = 6$. Solution set: $\{6\}$.

11c Write $\sqrt{2y - 5} = 1 + \sqrt{y - 3}$, then square and isolate the radical to get $2\sqrt{y - 3} = y - 3$. Square again and collect terms to obtain

$$y^2 - 10y + 21 = 0,$$

which has solutions 7 and 3. Both solutions are valid, and the solution set is $\{3, 7\}$.

11d We have $5x - 3 = \pm 12$, so $x = (3 \pm 12)/5$, and hence $x = -\frac{9}{5}, 3$. Solution set: $\{-\frac{9}{5}, 3\}$.

12 We have

$$\frac{A}{P} = (I + 1)^2 \quad \Rightarrow \quad \frac{P}{A} = \frac{1}{(1 + I)^2} \quad \Rightarrow \quad P = \frac{A}{(1 + I)^2}.$$

13a Either $2x - 4 \geq 8$ or $2x - 4 \leq -8$, and so either $x \geq 6$ or $x \leq -2$. Solution set: $(-\infty, -2] \cup [6, \infty)$.

13b This becomes $-4 < x + 6 < 4$, and hence $-10 < x < -2$. Solution set: $(-10, -2)$.

13c Either $3x + 1 > x - 2$ or $3x + 1 < -(x - 2)$, giving $x > -\frac{3}{2}$ or $x < \frac{1}{4}$. Solution set is all reals: $(-\infty, \infty)$.