Math 120 Exam #3 Key (Spring 2018)

1 Zeros are 0 (multiplicity 2), -1 (multiplicity 7), and 6 (multiplicity 1).

2 Find t such that $s(t) = 294$, or $4.9t^2 + 34.3t - 294 = 0$. It can help to multiply the equation by 10 to get

$$49t^2 + 343t - 2940 = 0,$$

and thus

$$t^2 + 7t - 60 = 0.$$

This factors as $(t-5)(t+12) = 0$, giving solutions $t = 5$ or $t = -12$. The answer must be $t = 5$ seconds.

3a 10 x-intercepts, at most.

3b 9 turning points, at most.

4

$$\begin{array}{r|rrrrr|r} 2 & 4 & 0 & 0 & -2 & 0 & 1 \\ & & 8 & 16 & 32 & 60 & 120 \\ \hline & 4 & 8 & 16 & 30 & 60 & 121 \end{array} \quad \longrightarrow \quad 4x^4 + 8x^3 + 16x^2 + 30x + 60 + \frac{121}{x-2}.$$

5 Possible rational zeros are $\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24$. Try 1.

$$\begin{array}{r|rrrr|r} 1 & 1 & -4 & -7 & 34 & -24 \\ & & 1 & -3 & -10 & 24 \\ \hline & 1 & -3 & -10 & 24 & 0 \end{array}$$

This shows 1 is a zero for f, and moreover

$$x^4 - 4x^3 - 7x^2 + 34x - 24 = (x-1)(x^3 - 3x^2 - 10x + 24).$$

Let $g(x) = x^3 - 3x^2 - 10x + 24$. Possible rational zeros are $\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24$. Trying 1 won't give a remainder of 0, but trying 2 does:

$$\begin{array}{r|rrr|r} 2 & 1 & -3 & -10 & 24 \\ & & 2 & -2 & -24 \\ \hline & 1 & -1 & -12 & 0 \end{array}$$

This shows 2 is a zero for g, and hence also for f. Moreover,

$$g(x) = (x-2)(x^2 - x - 12),$$

and so

$$f(x) = (x-1)g(x) = (x-1)(x-2)(x^2 - x - 12) = (x-1)(x-2)(x-4)(x+3).$$

This fully factors $f(x)$, and $f(x) = 0$ is seen to have solution set $\{-3, 1, 2, 4\}$.

6 The simplest choice is $(x+2)(x-3)^2(x+1)$, though $c(x+2)(x-3)^2(x+1)$ for any $c \neq 0$ works.

7 The additional zero $2+i$ will need to be included in order to have rational coefficients. Thus we have

$$f(x) = (x+1)[x-(2-i)][x-(2+i)] = (x+1)(x^2-4x+5) = x^3-3x^2+x+5.$$

8 Possible rational zeros are $\pm1, \pm2, \pm4$. Indeed -2 works:

$$\begin{array}{r|rrr|r} -2 & 1 & 0 & -2 & 4 \\ & & -2 & 4 & -4 \\ \hline & 1 & -2 & 2 & 0 \end{array}$$

Now we have $f(x) = (x+2)(x^2-2x+2)$. The trinomial x^2-2x+2 does not lend itself to direct factoring. Setting $x^2-2x+2=0$, use the quadratic formula or completing the square to get $x = 1 \pm i$. Thus the zeros of f are -2, $1-i$, $1+i$, and $f(x)$ factors as

$$f(x) = (x+2)[x-(1-i)][x-(1+i)].$$

9 For x in the domain of h we find that

$$h(x) = \frac{x-2}{x(x+6)},$$

and so the vertical asymptotes are $x=0$ and $x=-6$.

10a Factoring gives $(x-2)(x+1) \geq 0$, and so either $x \leq -1$ or $x \geq 2$. Solution set is $(-\infty,-1] \cup [2,\infty)$.

10b Move everything to one side:

$$x^5 - 2x^3 + x^2 - 2 < 0.$$

Factor by grouping, then factor the sum of cubes and difference of squares:

$$x^3(x^2-2) + (x^2-2) = (x^2-2)(x^3+1) = (x-\sqrt{2})(x+\sqrt{2})(x+1)(x^2-x+1) < 0.$$

The trinomial x^2-x+1 is never zero and so may be divided out to give

$$(x-\sqrt{2})(x+\sqrt{2})(x+1) < 0.$$

The solution set is $(-\infty,-\sqrt{2}) \cup (-1,\sqrt{2})$.

10c We have

$$\frac{x}{x-5} - 2 > 0 \quad \Rightarrow \quad \frac{10-x}{x-5} > 0.$$

Solution set is $(5,10)$.