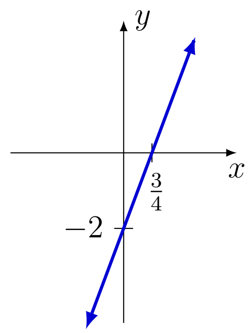
Math 120 Exam #1 Key (Spring 2018)

1a x-intercept: $(3/4, 0)$; y-intercept: $(0, -2)$.

1b



2 Distance is

$$\sqrt{\left(-\frac{11}{3}-\frac{1}{3}\right)^2+\left(-\frac{1}{2}-\frac{5}{2}\right)^2}=\sqrt{25}=5.$$

3 Center is $(4, -9)$, and radius is 11.

4 $f(-1) = -1, \quad f(-x) = 3 - 4x^2, \quad f(1-t) = 3 - 4(1-t)^2 = -4t^2 + 8t - 1.$

5a $\operatorname{Dom} f = (-\infty, -2/3) \cup (-2/3, \infty)$.

5b We have:

$$\begin{aligned}\operatorname{Dom} g &= \{x : x^2 + 4x - 21 \neq 0\} = \{x : (x+7)(x-3) \neq 0\} = \{x : x \neq -7, 3\} \\ &= (-\infty, -7) \cup (-7, 3) \cup (3, \infty).\end{aligned}$$

5c $\operatorname{Dom} h = \{x : 6x + 3 \geq 0\} = \left\{x : x \geq -\frac{1}{2}\right\} = \left[-\frac{1}{2}, \infty\right)$.

6 Domain is $(-\infty, \infty)$, and the range is $[-3, \infty)$.

7 Slope is

$$m = \frac{-5-(-13)}{-8-16} = -\frac{1}{3}.$$

8 Point-slope formula gives $y - 6 = -\frac{3}{8}(x-5)$, which becomes $y = -\frac{3}{8}x + \frac{63}{8}$.

9 Let x be the amount at 5% interest, so $9000 - x$ is the amount at 6% interest. We have:

$$0.05x + 0.06(9000 - x) = 492,$$

which becomes $-0.01x + 540 = 492$, and hence $x = 4800$. So \$4800 is borrowed at 5%, and \$4200 at 6%.

10a Inequalities becomes $4x^2 - 8x < 4x^2 - 14x + 6$, giving $-8x < -14x + 6$, and thus $x < 1$. Solution set: $(-\infty, 1)$.

10b Adding 6 gives $2 \leq 2x < 10$, and then dividing by 2 gives $1 \leq x < 5$. Solution set: $[1, 5)$.

10c We have $x < -\frac{4}{3}$ or $x > 2$. Solution set: $\left(-\infty, -\frac{4}{3}\right) \cup \left(2, \infty\right)$.

11a $\operatorname{Dom} f = [0, \infty)$ and $\operatorname{Dom} g = (-\infty, 3]$.

11b $\operatorname{Dom}(f - g) = \operatorname{Dom} f \cap \operatorname{Dom} g = [0, \infty) \cap (-\infty, 3] = [0, 3]$

11c $\operatorname{Dom}(ff) = \operatorname{Dom} f \cap \operatorname{Dom} f = \operatorname{Dom} f = [0, \infty)$.

11d $\operatorname{Dom}(f/g) = \{x : x \in \operatorname{Dom} f,\ x \in \operatorname{Dom} g,\ g(x) \neq 0\} = [0, 3)$.

12a We have

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) = f\left(\sqrt{x^2 - 36}\right) = 1 - \left(\sqrt{x^2 - 36}\right)^2 \\ &= 1 - (x^2 - 36) = 37 - x^2.\end{aligned}$$

and

$$(g \circ f)(x) = g(f(x)) = g(1 - x^2) = \sqrt{(1 - x^2)^2 - 36} = \sqrt{x^4 - 2x^2 - 35}$$

12b We have

$$\begin{aligned}\operatorname{Dom}(f \circ g) &= \{x : x \in \operatorname{Dom} g \text{ and } g(x) \in \operatorname{Dom} f\} \\ &= \left\{x : x \in (-\infty, -6] \cup [6, \infty) \text{ and } \sqrt{x^2 - 36} \in (-\infty, \infty)\right\} \\ &= (-\infty, -6] \cup [6, \infty),\end{aligned}$$

since $\sqrt{x^2 - 36} \in (-\infty, \infty)$ holds if and only if $x \in (-\infty, -6] \cup [6, \infty)$.

12c Noting that $1 - x^2 \in [6, \infty)$ is impossible for real x,

$$\begin{aligned}\operatorname{Dom}(g \circ f) &= \{x : x \in \operatorname{Dom} f \text{ and } f(x) \in \operatorname{Dom} g\} \\ &= \left\{x : x \in (-\infty, \infty) \text{ and } 1 - x^2 \in (-\infty, -6] \cup [6, \infty)\right\} \\ &= \left\{x : 1 - x^2 \in (-\infty, -6]\right\} \\ &= \left(-\infty, -\sqrt{7}\right] \cup \left[\sqrt{7}, \infty\right).\end{aligned}$$

13 We can let $g(x) = \sqrt[3]{3x + 7}$ and $f(x) = 1/x$. There are other possibilities.