Math 120 Exam #4 Key (Spring 2017)

1a Suppose $f(a) = f(b)$. Then $(a-60)^5 = (b-60)^5$, which implies $a-60 = b-60$, and thus $a = b$. Therefore f is one-to-one.

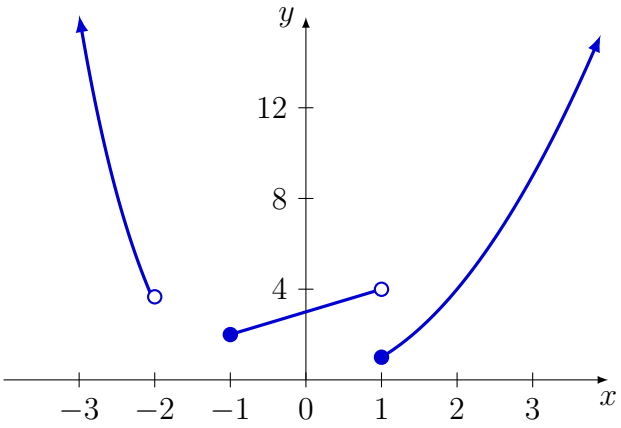
1b By definition $f(x) = y$ if and only if $f^{-1}(y) = x$. So, let $y = f(x)$, so that $y = (x-60)^5$. Then $x - 60 = \sqrt[5]{y}$, which implies that $x = 60 + \sqrt[5]{y}$, and therefore

$$f^{-1}(y) = 60 + \sqrt[5]{y}.$$

This is the formula for f^{-1}. (Replacing y with x in the formula is not necessary.)

2 Three examples are $f(x) = x$, $g(x) = 1/x$, and $h(x) = \sqrt{1-x^2}$ for $x \in [0,1]$. (The last example is the quarter of a unit circle in the first quadrant.)

3



4 $t^k = Q$ and $\log_p 3 = k$.

5 $\ln \sqrt[3]{ab^2} = \ln(ab^2)^{1/3} = \frac{1}{3}\ln(ab^2) = \frac{1}{3}\ln a + \frac{1}{3}\ln b^2 = \frac{1}{3}\ln a + \frac{2}{3}\ln b$.

6 We have

$$\ln x - \ln(x-5)^3 - \ln(x+5)^3 = \ln \frac{x}{(x-5)^3(x+5)^3} = \ln \frac{x}{(x^2-25)^3}$$

7a $4^{3x-5} = 16$ becomes $4^{3x-5} = 4^2$, so we must have $3x - 5 = 2$, and therefore $x = \frac{7}{3}$.

7b Convert to an exponential equation: $5^3 = 8 - 7x$, which implies $x = -\frac{117}{7}$.

7c Rewrite as

$$\log\left(\frac{2x+1}{x-2}\right) = 1 \quad \Rightarrow \quad \frac{2x+1}{x-2} = 10 \quad \Rightarrow \quad 2x+1 = 10x - 20,$$

and so $x = \frac{21}{8}$.

7d We have

$$\frac{e^x+e^{-x}}{e^x-e^{-x}}=3 \;\Rightarrow\; \frac{e^x+e^{-x}}{e^x-e^{-x}}\cdot\frac{e^x}{e^x}=3 \;\Rightarrow\; \frac{e^{2x}+1}{e^{2x}-1}=3 \;\Rightarrow\; e^{2x}+1=3e^{2x}-3,$$

giving $e^{2x}=2$. This implies that $\ln e^{2x}=\ln 2$, giving $2x=\ln 2$, and so $x=\frac{1}{2}\ln 2=\ln\sqrt{2}$.

7e Square both sides to get

$$\ln x=\left(\ln\sqrt{x}\right)^2=\left(\tfrac{1}{2}\ln x\right)^2=\tfrac{1}{4}(\ln x)^2,$$

and hence $(\ln x)^2-4\ln x=0$. Factoring gives

$$(\ln x)(\ln x-4)=0,$$

so that either $\ln x=0$ (so $x=1$) or $\ln x=4$ (so $x=e^4$). Solution set is $\{1,e^4\}$.

8 Population at time t (in years) is given by

$$P(t)=9,996,731(1.0108)^t,$$

and so we must find the time t when $P(t)=32,961,561,600$. We have

$$9{,}996{,}731(1.0108)^t=32{,}961{,}561{,}600 \;\Rightarrow\; (1.0108)^t=3297.2 \;\Rightarrow\; \ln(1.0108)^t=\ln 3297.2,$$

and hence

$$t=\frac{\ln 3297.2}{\ln 1.0108}\approx 754.1.$$

It will take about 754 years.

9 Add 5 times the 1st equation to the 2nd equation to get $25x=75$, or $x=3$. Now put $x=3$ into either equation and solve to get $y=-1$. Solution to the system: $(x,y)=(3,-1)$.

10 Add the 1st equation to the 3rd equation to obtain $2y=-4$, or $y=-2$. Put this result into the 1st, 2nd, and 4th equations to get the system

$$\begin{cases} x+\;\; z+\;\; w=4 \\ x+2z+4w=5 \\ -x+\;\; z-\;\; w=4 \end{cases} \tag{1}$$

For this system add the 1st and 3rd equations to get $2z=8$, or $z=4$. Put this result into the 1st and 2nd equations of the system (1) to get

$$\begin{cases} x+\;\; w=\;\; 0 \\ x+4w=-3 \end{cases}$$

This system solves to give $x=1$ and $w=-1$. The solution to the original system is therefore $(x,y,z,w)=(1,-2,4,-1)$.