

MATH 120 EXAM #3 KEY (SPRING 2017)

1 Zeros are 0 (multiplicity 3), 1 (multiplicity 5), and -4 (multiplicity 1).

2 Find t such that $s(t) = 294$, or $4.9t^2 + 34.3t - 294 = 0$. It can help to multiply the equation by 10 to get

$$49t^2 + 343t - 2940 = 0,$$

and thus

$$t^2 + 7t - 60 = 0.$$

This factors as $(t - 5)(t + 12) = 0$, giving solutions $t = 5$ or $t = -12$. The answer must be $t = 5$ seconds.

3a 10 x -intercepts, at most.

3b 9 turning points, at most.

4

$$\begin{array}{r|rrrr} -3 & 4 & 0 & 0 & -2 & 5 \\ & & -12 & 36 & -108 & 330 \\ \hline & 4 & -12 & 36 & -110 & 335 \end{array} \longrightarrow 4x^3 - 12x^2 + 36x - 110 + \frac{335}{x+3}.$$

5 Possible rational zeros are $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$. Try 1.

$$\begin{array}{r|rrrr} 1 & 1 & -4 & -7 & 34 & -24 \\ & & 1 & -3 & -10 & 24 \\ \hline & 1 & -3 & -10 & 24 & 0 \end{array}$$

This shows 1 is a zero for f , and moreover

$$x^4 - 4x^3 - 7x^2 + 34x - 24 = (x - 1)(x^3 - 3x^2 - 10x + 24).$$

Let $g(x) = x^3 - 3x^2 - 10x + 24$. Possible rational zeros are $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$. Trying 1 won't give a remainder of 0, but trying 2 does:

$$\begin{array}{r|rrr} 2 & 1 & -3 & -10 & 24 \\ & & 2 & -2 & -24 \\ \hline & 1 & -1 & -12 & 0 \end{array}$$

This shows 2 is a zero for g , and hence also for f . Moreover,

$$g(x) = (x - 2)(x^2 - x - 12),$$

and so

$$f(x) = (x - 1)g(x) = (x - 1)(x - 2)(x^2 - x - 12) = (x - 1)(x - 2)(x - 4)(x + 3).$$

This fully factors $f(x)$, and $f(x) = 0$ is seen to have solution set $\{-3, 1, 2, 4\}$.

6 The simplest choice is $(x + 2)(x - 3)^2(x + 1)$, though $c(x + 2)(x - 3)^2(x + 1)$ for any $c \neq 0$ works.

7 The additional zero $2 + i$ will need to be included in order to have rational coefficients. Thus we have

$$f(x) = (x + 1)[x - (2 - i)][x - (2 + i)] = (x + 1)(x^2 - 4x + 5) = x^3 - 3x^2 + x + 5.$$

8 Possible rational zeros are $\pm 1, \pm 2, \pm 4$. Indeed -2 works:

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -2 & 4 \\ & & -2 & 4 & -4 \\ \hline & 1 & -2 & 2 & 0 \end{array}$$

Now we have $f(x) = (x + 2)(x^2 - 2x + 2)$. The trinomial $x^2 - 2x + 2$ does not lend itself to direct factoring. Setting $x^2 - 2x + 2 = 0$, use the quadratic formula or completing the square to get $x = 1 \pm i$. Thus the zeros of f are $-2, 1 - i, 1 + i$, and $f(x)$ factors as

$$f(x) = (x + 2)[x - (1 - i)][x - (1 + i)].$$

9 For x in the domain of h we find that

$$h(x) = \frac{x - 2}{x(x + 6)},$$

and so the vertical asymptotes are $x = 0$ and $x = -6$.

10a Factoring gives $(x - 2)(x + 1) \geq 0$, and so either $x \leq -1$ or $x \geq 2$. Solution set is $(-\infty, -1] \cup [2, \infty)$.

10b Move everything to one side:

$$x^5 - 2x^3 + x^2 - 2 < 0.$$

Factor by grouping, then factor the sum of cubes and difference of squares:

$$x^3(x^2 - 2) + (x^2 - 2) = (x^2 - 2)(x^3 + 1) = (x - \sqrt{2})(x + \sqrt{2})(x + 1)(x^2 - x + 1) < 0.$$

The trinomial $x^2 - x + 1$ is never zero and so may be divided out to give

$$(x - \sqrt{2})(x + \sqrt{2})(x + 1) < 0.$$

The solution set is $(-\infty, -\sqrt{2}) \cup (-1, \sqrt{2})$.

10c We have

$$\frac{x}{x - 5} - 2 > 0 \Rightarrow \frac{10 - x}{x - 5} > 0.$$

Solution set is $(5, 10)$.