Math 120 Exam #2 Key (Spring 2017)

1a Since
$$f(-x) = 7 - 3(-x)^2 + 19(-x)^4 = 7 - 3x^2 + 19x^4 = f(x),$$
the function f is even.

1b Here $g(-x) = 2(-x) - |-x| = -2x - |x|$, which equals neither $g(x)$ nor $-g(x)$, and so the function is neither even nor odd.

2a $f(x) = \sqrt{x+6} - 2$

2b $f(x) = -(x-3)^2 + 8$

3a $7i \cdot 3i = 21i^2 = -21.$

3b $18i - 15i^2 = 18i + 15 = 15 + 18i.$

3c $(5-4i)(5-4i) = 25 - 40i + 16i^2 = 9 - 40i.$

3d $\dfrac{i}{2+i} \cdot \dfrac{2-i}{2-i} = \dfrac{2i - i^2}{4 - i^2} = \dfrac{2i+1}{5} = \dfrac{1}{5} + \dfrac{2}{5}i.$

4 Factoring gives $t(3t-2)(t-1) = 0$, and so the solution set is $\{0, 1, \frac{2}{3}\}$.

5 We have
$$x^2 - 8x = -9 \quad\Rightarrow\quad x^2 - 8x + 16 = -9 + 16 \quad\Rightarrow\quad (x-4)^2 = 7 \quad\Rightarrow\quad x - 4 = \pm\sqrt{7},$$
and so the solution set is $\{4 - \sqrt{7}, 4 + \sqrt{7}\}$.

6 The quadratic formula gives
$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)} = \frac{1 \pm \sqrt{-3}}{2} = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i.$$

7 Factoring yields:
$$(y^2 - 16)(y^2 + 1) = 0,$$
so either $y^2 - 16 = 0$ (giving the solutions ± 4) or $y^2 + 1 = 0$ (giving the solutions $\pm i$). Solution set is $\{-4, 4, -i, i\}$.

8 Let ℓ be the length and w the width. We're given $2\ell + 2w = 28$ and $\ell w = 48$, which implies $(14 - w)w = 48$, and hence $(w-6)(w-8) = 0$. Thus we find that $w = 6, 8$. When $w = 6$ we get $\ell = 8$, and vice-versa. Therefore the dimensions are 6 ft by 8 ft.

9a Get into vertex form:
$$f(x) = -3(x^2 - 8x) - 49 = -3(x^2 - 8x + 16) - 49 - (-3)(16) = -3(x-4)^2 - 1.$$

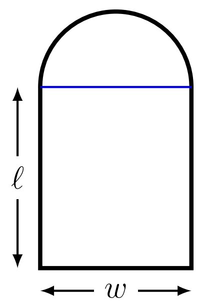
So f has vertex at $(4, -1)$.

9b Maximum value is $f(4) = -1$.

9c Range: $(-\infty, -1]$.

9d Increasing on $(-\infty, 4)$, decreasing on $(4, \infty)$.

10 Let w and ℓ be the width and length of the rectangular part of the window, as in the figure:



(Note: the "outer edges" do not include the top side of the rectangle!) The radius of the semicircle is then $\frac{1}{2}w$. The perimeter is to be 24 ft, so

$$24 = 2\ell + w + \frac{\pi}{2}w,$$

where $\frac{\pi}{2}w$ is half the circumference of a circle of radius $\frac{1}{2}w$. Now,

$$\ell = 12 - \frac{\pi + 2}{4}w.$$

Let $A(w)$ be the function that gives the area of the window as a function of w. We have

$$A(w) = w\ell + \tfrac{1}{2}\pi\left(\tfrac{1}{2}\right)^2 = w\left(12 - \frac{\pi + 2}{4}w\right) + \frac{\pi}{8}w^2 = -\frac{\pi + 4}{8}w^2 + 12w.$$

We want to find w for which $A(w)$ is maximal. To do this either use a formula or get vertex form:

$$A(w) = -\frac{\pi + 4}{8}\left(w - \frac{48}{\pi + 4}\right)^2 + \frac{288}{\pi + 4}.$$

Thus $A(w)$ is maximal when $w = 48/(\pi + 4)$. Length is then

$$\ell = 12 - \frac{\pi + 2}{4}w = 12 - \frac{\pi + 2}{4} \cdot \frac{48}{\pi + 4} = \frac{24}{\pi + 4}.$$

Dimensions of the rectangular part: $48/(\pi + 4)$ ft wide and $24/(\pi + 4)$ ft high.

11a Multiply by $x^2 - 9$ to get

$$2 + 5(x + 3) = 3(x - 3) \quad \Rightarrow \quad x = -13.$$

Solution set: $\{-13\}$.

11b Square both sides once, and then a second time:

$$\left(\sqrt{x+4}+2\right)^2 = x^2 \quad\Rightarrow\quad 4\sqrt{x+4} = x^2 - x - 8$$
$$\Rightarrow\quad 16(x+4) = x^4 - 2x^3 - 15x^2 + 16x + 64$$
$$\Rightarrow\quad x^4 - 2x^3 - 15x^2 = 0,$$

and so we have $x^2(x-5)(x+3) = 0$, and finally $x = 0, 5, -3$. However 0 and -3 are extraneous solutions. The solution set is $\{5\}$.

11c A square root is never equal to a negative number, so the left-hand side of the equation cannot possibly equal 0 for any value of x. There is thus no solution. Solution set: $\varnothing$.

11d We have $4x - 3 = \pm 11$, so $x = (3 \pm 11)/4$, and hence $x = \frac{7}{2}, -2$. Solution set: $\{\frac{7}{2}, -2\}$.

12 $t = \dfrac{ab}{a+b}$.

13a We have

$$-13 < 3x + 4 < 13 \quad\Rightarrow\quad -\frac{17}{3} < x < 3.$$

Solution set: $\left(-\frac{17}{3}, 3\right)$.

13b Either $x + 6 \geq 4$ or $x + 6 \leq -4$, and so $x \geq -2$ or $x \leq -10$. Solution set in interval notation: $(-\infty, -10] \cup [-2, \infty)$.

13c Either $3x - 1 > 5x + 2$ or $3x - 1 < -(5x + 2)$, giving $x < -\frac{3}{2}$ or $x < -\frac{1}{8}$. Solution set: $\left(-\infty, -\frac{1}{8}\right)$.