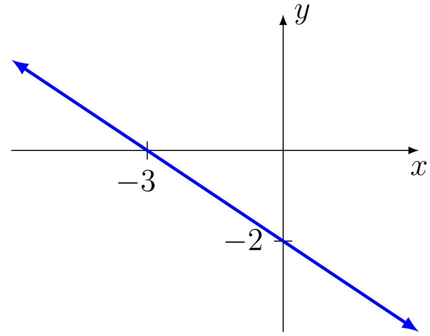
MATH 120 EXAM #1 KEY (SPRING 2017)

1a x-intercept: $(-3, 0)$; y-intercept: $(0, -2)$.

1b



2 Distance is

$$\sqrt{\left(-\tfrac{11}{3} - \tfrac{1}{3}\right)^2 + \left(-\tfrac{1}{2} - \tfrac{5}{2}\right)^2} = \sqrt{25} = 5.$$

3 Center is $(-1, 2)$, and radius is 8.

4 $f(-1) = -1, \quad f(-x) = 3 - 4x^2, \quad f(1-t) = 3 - 4(1-t)^2 = -4t^2 + 8t - 1.$

5a $\text{Dom}\, f = (-\infty, 4) \cup (4, \infty)$.

5b We have:

$$\begin{aligned}\text{Dom}\, g &= \{x : x^2 - 2x - 15 \neq 0\} = \{x : (x-5)(x+3) \neq 0\} = \{x : x \neq -3, 5\} \\ &= (-\infty, -3) \cup (-3, 5) \cup (5, \infty).\end{aligned}$$

5c $\text{Dom}\, h = \{x : 2 - 3x \geq 0\} = \left\{x : x \leq \tfrac{2}{3}\right\} = \left(-\infty, \tfrac{2}{3}\right]$.

6 Domain is $(-\infty, \infty)$, and the range is $[-3, \infty)$.

7 Slope is

$$m = \frac{-5 - (-13)}{-8 - 16} = -\frac{1}{3}.$$

8 Point-slope formula gives $y - 6 = -\frac{3}{8}(x - 5)$, which becomes $y = -\frac{3}{8}x + \frac{63}{8}$.

9 Let x be the amount at 5% interest, so $9000 - x$ is the amount at 6% interest. We have:

$$0.05x + 0.06(9000 - x) = 492,$$

which becomes $-0.01x + 540 = 492$, and hence $x = 4800$. So \$4800 is borrowed at 5%, and \$4200 at 6%.

10a Inequalities becomes $4x^2 - 8x < 4x^2 - 14x + 6$, giving $-8x < -14x + 6$, and thus $x < 1$. Solution set: $(-\infty, 1)$.

10b Subtracting 6 gives $-10 \leq -2x < -2$, and then dividing by -2 gives $5 \geq x > 1$. Solution set: $(1, 5]$.

10c We have $x < -\frac{4}{3}$ or $x > 2$. Solution set: $\big(-\infty, -\frac{4}{3}\big) \cup \big(2, \infty\big)$.

11a $\operatorname{Dom} f = [0, \infty)$ and $\operatorname{Dom} g = (-\infty, 2]$.

11b $\operatorname{Dom}(f - g) = \operatorname{Dom} f \cap \operatorname{Dom} g = [0, \infty) \cap (-\infty, 2] = [0, 2]$

11c $\operatorname{Dom}(ff) = \operatorname{Dom} f \cap \operatorname{Dom} f = \operatorname{Dom} f = [0, \infty)$.

11d $\operatorname{Dom}(f/g) = \{x : x \in \operatorname{Dom} f,\ x \in \operatorname{Dom} g,\ g(x) \neq 0\} = [0, 2)$.

12a We have

$$\begin{aligned}(f \circ g)(x) = f(g(x)) = f\big(\sqrt{x^2 - 36}\,\big) &= 1 - \big(\sqrt{x^2 - 36}\,\big)^2 \\ &= 1 - (x^2 - 36) = 37 - x^2.\end{aligned}$$

and

$$(g \circ f)(x) = g(f(x)) = g(1 - x^2) = \sqrt{(1 - x^2)^2 - 36} = \sqrt{x^4 - 2x^2 - 35}$$

12b We have

$$\begin{aligned}\operatorname{Dom}(f \circ g) &= \{x : x \in \operatorname{Dom} g \text{ and } g(x) \in \operatorname{Dom} f\} \\ &= \big\{x : x \in (-\infty, -6] \cup [6, \infty) \text{ and } \sqrt{x^2 - 36} \in (-\infty, \infty)\big\} \\ &= (-\infty, -6] \cup [6, \infty),\end{aligned}$$

since $\sqrt{x^2 - 36} \in (-\infty, \infty)$ holds if and only if $x \in (-\infty, -6] \cup [6, \infty)$.

12c Noting that $1 - x^2 \in [6, \infty)$ is impossible for real x,

$$\begin{aligned}\operatorname{Dom}(g \circ f) &= \{x : x \in \operatorname{Dom} f \text{ and } f(x) \in \operatorname{Dom} g\} \\ &= \big\{x : x \in (-\infty, \infty) \text{ and } 1 - x^2 \in (-\infty, -6] \cup [6, \infty)\big\} \\ &= \big\{x : 1 - x^2 \in (-\infty, -6]\big\} \\ &= \Big(-\infty, -\sqrt{7}\Big] \cup \Big[\sqrt{7}, \infty\Big).\end{aligned}$$

13 We can let $g(x) = \sqrt[3]{3x + 7}$ and $f(x) = 1/x$. There are other possibilities.